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Master Thesis

Use of option pricing method for a company valuation

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Declaration

I declare that I have created this Master Thesis by myself and that I have used only resources which I quote and which are stated in the enclosed list of used resources.

In Brno, at 20th May 2007 ……………………………………

I would like to thank to the tutor of my Master Thesis, Prof. Ing. Libor Grega, Ph.D. for the expert conduct and for the help provided by processing of the Master Thesis. My sincere thanks belong also to consultant Ing. Boris Mišun from the company Ernst & Young for precious advices and willingness by providing me the information necessary for this thesis.

Abstract

This thesis deals with the calculation of the value of the firm on the basis of option pricing methodology. This methodology is applied on the specific company working in the armament industry.

The first part of the thesis focuses on the brief description of traditional valuation methods, definition of options and detailed description of theory of option pricing methods. The second part is dedicated to application of the option pricing methodology, more precisely of the trinomial method, for valuation of the really existing Czech company.

Abstrakt

Diplomová práce se zabývá kalkulací hodnoty firmy na základě opční metodologie. Tato metodologie je aplikována na specifickou společnost působící ve zbrojařském průmyslu.

První část práce se zaměřuje na krátký popis tradičních oceňovacích metod, definování opcí a detailní popis teorie týkající se opční metodologie. Druhá část práce je věnována aplikaci opční metodologie, přesněji tedy trinomické metody, na ocenění skutečně existující české společnosti.

Content

1 Introduction

In the last few years new phenomenon has arised. The risks arising from the world terorism, accompanied by suicidal bomb attacks, fears from use of chemical and biological weapons or computer attacks, create the need for affected states to defend against this threat.

The situation between the "rich North and poor South" accompanied by the fanatism and fight of the Great Powers for natural resources will be continually worsening. Such climate creates the basis for existence of companies producing the weapons and military technologies and equipment.

The value of such companies is unimaginable for most of us and the valuation itself conceals the fact that there can exist neither one right valuation, which could be applicable in all cases.

The methodology of real options for evaluation of a firm in conditions of the Czech Republic is not widespread. The metodology of real options was developed in the USA and there was not issued any complete publication concerning this methodology in the Czech Republic so far. However, the application of this methodolgy is wholly feasible, therefore it is desirable to investigate it and deepen it further.

2 Aim of the Master Thesis and the methodology used

The aim of this Master Thesis is to calculate the value of the firm on the basis of option pricing methodology. In the practical part this methodology will be applied on the specific company working in the armaments industry.

The Master Thesis is structurated into diverse chapters and subchapters for better lucidity. The first section consisting of chapters 3, 4 and 5 has a character of literary search. The third chapter is focused on brief description of classic valuation methods, which are further used as a basis for calculation of option pricing methodology. Further in fourth chapter the attention will be paid to options, to their division and to possibilities of their use.

The content of the fifth chapter are option pricing models, their description and assumptions of their use when evaluating the company as a real option.

The theoretical part of the Master Thesis is based on the theories of various authors. Information used in the Master Thesis were gathered from technical bibliography concerning the topic issues, from internal information of the valued company, from Internet and from consultations with valuation experts. The full list of the literature used is stated at the end of the thesis.

The sixth chapter is the practical part and it is dedicated to the application of option pricing methodology itself to calculate the value of certain company.

The point of departure in the calculation are earnings before interest and taxes, so called EBIT. First of all the development of EBIT in the past five years has to be examined. For the analysis of past evolution, the trend analysis is used to determine the tendency of future development. For the calculation are used following equations [7]:

$$b = \frac{\sum t y_t - \bar{t} \sum y_t}{\sum t^2 - n \bar{t}^2},$$

$$a = \bar{y} - b\bar{t} \ ,$$

$est\ T_t = a + bt$.

where *a* and *b* are the unknown parameters of trend line T_t, *y* is the time series variable and *t* = 1,2, …, n is the time variable. Parameter *a* states the value of the trend in time *t* = 0, and parameter *b* expresses the change of trend when the value of time variable increases by 1 unit.

Subsequently, the analysis of dependence between adjacent members of one time series has to be made. The degree of closeness of this dependence is the autocorrelation coefficient of first order r_1. For the calculation of autocorrelation coefficient the following formula is used [7]:

$$r_1 = \frac{(n-1)\sum_{t=1}^{n-1} y_t y_{t+1} - \sum_{t=1}^{n-1} y_t \sum_{t=1}^{n-1} y_{t+1}}{\sqrt{\left[(n-1)\sum_{t=1}^{n-1} y_t^2 - \left(\sum_{t=1}^{n-1} y_t\right)^2\right]\left[(n-1)\sum_{t=1}^{n-1} y_{t+1}^2 - \left(\sum_{t=1}^{n-1} y_{t+1}\right)^2\right]}} .$$

In the case, the autocorrelation coefficient is of low value, the prediction of future development of EBIT has to be made on the basis of expert estimation and on the basis of budget and business plan for future five years, which was obtained from the company’s management.

The methodology stated above is used also for the prediction of future development of depreciation, future investments and change in net working capital.

Subsequently the free cash flow has to be calculated for each node of the trinomial tree according to equation [2]:

FCFF = EBIT (1 - tax rate) + Depreciation - Capital Expenditure – change of Working Capital.

After this the weighted average cost of capital has to be calculated according to equation [8]:

WACC = cost of equity (equity / (debt + equity)) + cost of debt (debt / (debt + equity))

Cost of equity R_A is calculated according to CAPM model as [13]:

$$R_A = R_F + (E(R_M) - R_F) \cdot \beta_A,$$

where R_F is the risk free rate, which is determined as a interest rate of 15-year Treasury Bonds; $(E(R_M) - R_F)$ is the risk premium and β_A is beta coefficient.

The beta coefficient is calculated based on the equation [13]:

$$\beta_L = \beta_u \left(1 + (1-t)\left(\frac{D}{E}\right)\right),$$

where β_L is levered beta for equity in the firm, β_u is unlevered beta of the firm (beta of the firm without any debt), t is corporate tax rate and D/E is debt to equity ratio.

Cost of debt has to be calculated as a weighted average of various interest rate of various debts of the company.

Subsequently it is necessary to compute the present value of the free cash flow. It is assumed that the company will exist infinitely long time and therefore the present value of FCFF is determined as a present value of infinite sequence of FCFF discounted at a weighted average cost of capital [2]:

$$PV\ of\ FCFF = \frac{FCFF_t \cdot (1+\pi)^n}{WACC}.$$

The FCFF is augmented for the inflation rate π each year.

Afterwards, the estimation of the future development of company's debt has to be made on the basis of expert estimation and business plan provided by the company management for next five years.

On the basis of values calculated according to the procedures described above, the intrinsic value of the call option has to be calculated for individual nodes of trinomial tree according to the relation [8]:

$$Intrinsic\ value\ of\ \ call\ option = \max(S - K, 0),$$

where S is the present value of the underlying asset (in this case it is the present value of FCFF) and K is the strike price of the option, which in practise means the value of the debt.

And finally the calculation of the value of the equity as an American call option is determined for individual nodes of trinomial tree as follows [6]:

$$C_t = \max(PV[E(C_{t+1})]; Instrinsic\ value\ of\ \ call\ option).$$

where it is proceeded from the end of the trinomial tree. The expression $PV[E(C_{t+1})]$ represents the present value of expected value of the option in subsequent period and is determined as [6]:

$$PV[E(C_{t+1})] = (1+r)^{-1} \cdot [C^{u}_{t+1} \cdot p_i + C^{d}_{t+1} \cdot q_i + (1 - p_i - q_i) \cdot C^{m}_{t+1}],$$

where r is the risk-free interest rate, p_i is the probability of up movement, q_i is the probability of down movement, $C_{t+1}{}^{u}$ is the intrinsic value of up movement, $C_{t+1}{}^{d}$ is the intrinsic value of down movement and $C_{t+1}{}^{m}$ is the intrinsic value when there is no change.

3 Basis and significance of the valuation

In general, valuation can be defined as a complex of methods and procedures, which lead to determination of the market value of a firm. The market value of the company is then the estimated amount for which the asset should be traded.

We should be aware of the difference between the value and the price of an asset. While the value of the asset represents the economic concept concerning the financial relationship between the assets, the price is offered, desired and paid amount for the asset.

Every asset, financial as well as real, has a value. For the successful investment and management of these assets we need to understand not only what the value is but also the sources of the value. Any asset can be valued, however some assets are easier to value than others and the details of valuation will vary from case to case.

The value obtained from any valuation model is affected by firm-specific as well as market-wide information. As a consequence, the value of the asset will change as new information appears. If we consider the constant flow of information into financial markets, a valuation of a firm dates rapidly and has to be updated to reflect the current information. Such information can be specific for the company, can affect the whole sector or change expectations for all subjects in the market. There will be always uncertainty about the result, even at the end of the most careful and detailed valuation, as the valuation is made on the basis of assumptions about the future of the company and the economy. It is unrealistic to expect absolute certainty in valuation as the cash flows and discount rates are estimated with error.

Valuation plays a key role in many areas of finance - in corporate finance, mergers and acquisitions and portfolio management. Analysts use a wide range of models to value assets in practice, ranging from the simple to the sophisticated. These models often make very different assumptions about pricing, but they do share some common characteristics. However, there exists neither one valuation model which could be used in all cases and for all purposes. Therefore it is essential to know for which purpose the valution is made.

The reasons for the valuation can be divided into two groups:

a) Valuation depending on the will of the owners – bound to the business and economic requirements (e.g. purchase and sale of the firm, consolidation).

b) Valuation non depending on the will of the owners – bound to the legal regulations (e.g. required for fiscal or divorce proceeding).

The most common reasons for valuation can be summarized into following points:

- purchase and sale of the company,
- change of the legal form,
- request for a loan,
- fusion and consolidation of the companies,
- payment of taxes,
- initial deposit of the company into newly created company,
- introduction on the stock market,
- decision making about the financial rehabilitation.

3.1 Traditional valuation methods

In general terms, there are three approaches to valuation. The first, discounted cashflow valuation relates the value of an asset to the present value of expected future cashflows on that asset. The second, relative valuation, estimates the value of an asset by looking at the pricing of comparable assets relative to a common variable such as earnings, cashflows, book value or sales. The third, contingent claim valuation uses option pricing models to measure the value of assets that share option characterictics.

In this section will be decribed the principles of discounted cashflow valuation, which is one of the traditional approaches. The contingent claim valuation will be discussed more deeply afterward.

3.1.1 Discounted Cash Flow valuation

The discounted cash flow valuation is one of the three ways of approaching the income valuation. To apply option pricing models to value assets, we often have to begin with a discounted cash flow valuation. This is why we focuse on discounted cash flow valuation in this section.

This approach has its foundation in the present value rule, where the value of any asset is the present value of expected future cashflows that the asset generates [8]:

$$Value = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t}, \tag{3.1}$$

where

n = life of the asset,

CF_t = cash flow in period t,

r = discount rate reflecting the riskiness of the estimated cashflows.

The cashflows will vary from asset to asset - dividends for stocks, coupons (interest) and the face value for bonds and after-tax cashflows for a real project.

The value of the firm is obtained by discounting expected cashflows to the firm, i.e., the residual cashflows after meeting all operating expenses, reinvestment needs and taxes, but prior to any payments to either debt or equity holders, at the weighted average cost of capital, which is the cost of the different components of financing used by the firm, weighted by their market value proportions [8]:

$$Value\ of\ \ firm = \sum_{t=1}^{t=n} \frac{FCFF_t}{(1+WACC)^t}, \tag{3.2}$$

where

$FCFF_t$ = expected cashflow to firm in period t,

$WACC$ = weighted average cost of capital,

n = life of the asset.

The weighted average cost of capital is calculated as [8]

$$WACC = \cos t\ of\ equity \cdot \left(\frac{equity}{debt + equity} \right) + \cos t\ of\ debt \cdot \left(\frac{debt}{debt + equity} \right). \qquad (3.3)$$

The formula (3.2) stated above assumes the constant growth of free cash flow. It is designed to value firms that are growing at a stable rate and are hence in a steady state. It is therefore better to use multi-stage models, which divide the time of existence of a company in several stages.

The most common is the ***two-stage FCFF model***. It is designed to value a firm which is expected to grow much faster than a stable firm in the initial period and at a stable rate after that.

The value of any firm is the present value of the FCFF per year for the extraordinary growth period plus the present value of the continuing value in the second period.

If the firm reaches steady state after *n* years and starts growing at a stable growth rate g_n after that, the value of the firm can be written as [8]:

$$Value\ of\ \ firm = \sum_{t=1}^{t=n} \frac{FCFF_t}{(1+WACC)^t} + \frac{\left[FCFF_{n+1} / (WACC - g_n) \right]}{(1+WACC)^n}. \qquad (3.4)$$

The growth rate used in the model has to be reasonable, relative to the nominal growth rate in the economy in which the firm operates. As a general rule, a “stable” growth rate cannot exceed the growth rate of the economy in which the company operates by more than one or two percent. This model is best suited for firms which are growing at a rate comparable to or lower than the nominal growth in the economy.

A firm that is growing at a rate that it can sustain in perpetuity – a stable growth rate – can be valued using a ***stable growth model***.

A firm with free cash flow growing at a stable rate growth rate can be valued using the following equation [8]:

$$Value\ of\ \ firm = \frac{FCFF_1}{WACC - g_n}, \tag{3.5}$$

where

$FCFF_1$ = Expected FCFF next year,

$WACC$ = Weighted average cost of capital,

g_n = Growth rate in the FCFF (forever).

The free cash flow to the firm is the sum of the cash flows to all claim holders in the firm, including stockholders, bondholders and preffered stockholders. There are two ways of measuring the free cash flow to the firm *FCFF*.

One is to add up the cashflows to the claim holders, which would include cash flows to equity (defined either as free cash flow to equity or dividends), cashflows to lenders (which would include principal payments, interest expenses and new debt issues) and cash flows to preferred stockholders (usually preferred dividends) [2]:

FCFF = Free Cashflow to Equity + Interest Expense (1 - tax rate) + Principal Repayments - New Debt Issues + Preferred Dividends (3.6)

A simpler way of getting to free cash flow to the firm is to estimate the cash flows prior to any of these claims. Thus, we could begin with the earnings before interest and taxes, net out taxes and reinvestment needs and arrive at an estimate of the free cash flow to the firm [2]:

FCFF = EBIT (1 - tax rate) + Depreciation - Capital Expenditure – change of Working Capital (3.7)

Since this cash flow is prior to debt payments, it is often referred to as an unlevered cash flow. Note that this free cash flow to the firm does not incorporate any of the tax benefits due to interest payments. This is because the use of the after-tax cost of debt in the cost of capital already considers this benefit and including it in the cash flows would double count it.

As was stated above, the weighted average cost of capital is weighted average of cost equity and of cost of debt. The cost of equity can be determined according to the ***Capital asset pricing model (CAPM)***.

The capital asset pricing model is an economic model for valuing stocks, securities, derivatives and assets by relating risk and expected return. The model is based on the idea that investors demand additional expected return (so called risk premium) if they are asked to accept additional risk. The CAPM model says that this expected return that the investors would demand is equal to the rate on a risk-free security plus a risk premium. If the expected return does not meet the required return, the investors will refuse to invest.

The capital asset pricing model is valid only within these set of assumptions [13]:

- Investors are risk averse individuals who maximize the expected utility of their final wealth.
- Investors have homogenous expectations about asset returns.
- Asset returns are distributed by the normal distribution.
- There exists a risk-free asset and investors may borrow or lend unlimited amounts of this asset at a constant rate – the risk free rate.
- There is a definite number of assets and their quantities are fixed within the one period world.
- All assets are perfectly divisible and priced in a perfectly competitive market.
- Asset markets are frictionless and information is costless and simultaneously available to all investors.
- There are no market imperfections such as taxes, regulations or restrictions on short selling.

The expected return of an asset can be written as a function of the risk-free rate and the beta coefficient of that asset [13]:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f) \tag{3.8}$$

where

$E(R_i)$ = expected return on asset *i*,

R_f = risk-free rate,

$E(R_m)$ = expected return on market portfolio,

β_i = beta of investment *i*.

The riskless asset is defined to be an asset for which the investor knows the expected return with certainty for the time horizon of the analysis. The risk premium is the premium demanded by investors for investing in the market portfolio, which includes all risky assets in the market, instead of investing in a riskless asset.

Beta measures the risk of a firm relative to a market index. The more sensitive a business is to market conditions, the higher is beta.

If all the firm's risk is borne by the stockholders and debt has a tax benefit to the firm, then [8]:

$$\beta_L = \beta_u \left(1 + (1-t)\left(\frac{D}{E}\right)\right), \tag{3.9}$$

where

β_L = levered beta for equity in the firm,

β_u = unlevered beta of the firm (beta of the firm without any debt),

t = corporate tax rate,

D/E = debt/equity ratio.

The tax factor in the equation measures the tax deductibility of interest payments. According to *DAMODARAN (2007)* the ***unlevered beta*** of a firm is determined by the types of the businesses in which it operates and its operating leverage. It is often also referred to as the asset beta since it is determined by the assets owned by the firm. Thus the ***levered beta***, which is also the beta for an equity investment in a firm or the equity beta, is determined both by the riskiness of the business it operates in and by the amount of financial leverage risk it has taken on.

The unlevered beta of industry is estimated by unlevering the levered beta of industry by the industry debt to equity ratio [8]:

$$\beta_{u\ industry} = \frac{\beta_{L\ industry}}{1+(1-t)(industry\ D/E)}\ . \qquad (3.10)$$

3.2 Other valuation methods

Expect of traditional valuation methods, there exist other approaches which were mentioned above. In this section will be superficially described methods such as relative valuation method and method based on EVA. Other methods will not be mentioned, as they are not subject of the master thesis.

3.2.1 Relative valuation

In relative valuation, the value of an asset is derived from the pricing of “comparable” assets, standardized using a common variable such as earnings, cashflows, book value or revenues. For relative valuation can be used an industry-average price-earnings ratio to value a firm. It is assumed that the other companies in the industry are comparable to the company which is being valued and that the market prices these companies correctly. Another multiple which is widely used is the price to book value ratio, with companies selling at a discount on book value, relative to comparable companies, being considered undervalued [2].

3.2.2 Method based on the Economic Value Added (EVA)

The Economic Value Added (EVA) is a measure of surplus value created on an investment or a portfolio of investments. It is computed as the product of the "excess return" made on an investment and the capital invested in that investment [8]:

EVA = (Return on Capital Invested – Cost of Capital)(Capital Invested) = After tax operating income – (Cost of Capital)(Capital invested) (3.11)

The basis of the valuation based on the EVA is the net present value rule. The net present value (NPV) of a project, which reflects the present value of expected cash flows on a project, netted against any investment needs, is a measure of surplus value on the project. Thus investing in projects with positive net present value will increase the value of the firm, while investing in projects with negative net present value will reduce value. Economic value added is a simple extension of the net present value rule. The net present value of the project is the present value of the economic value added by that project over its life [8]:

$$NPV = \sum_{t=1}^{t=n} \frac{EVA_t}{(1+k_c)^t}, \qquad (3.12)$$

where EVA_t is the economic value added by the project in year t and the project has a life of n years.

Such connection between the economic value added and NPV allows to link the value of a firm to the economic value added by that firm [8]:

$$Firm\,Value = Capital\ invested_{Assets\ in\ place} + NPV_{Assets\ in\ place} + \sum_{t=1}^{t=\infty} NPV_{Future\ projects,t} \qquad (3.13)$$

Substituting the economic value added version of net present value into this equation, we get

$$Firm\,Value = Capital\ invested_{Assets\ in\ place} + \sum_{t=1}^{t=\infty} \frac{EVA_{t,Assets\ in\ place}}{(1+k_c)^t} + \sum_{t=1}^{t=\infty} \frac{EVA_{t,Future\ projects}}{(1+k_c)^t} \qquad (3.14)$$

Thus the value of a firm can be written as the sum of three components – the capital invested in assets in place, the present value of the economic value added by these assets and the expected present value of the economic value that will be added by future investments.

4 Options and methods of valuation

An option is a derivative, which can be defined as financial instrument whose value depends on, or derives from the values of other, more basic underlying variables. In most cases the variable underlying derivative is the price of traded asset. A stock option for example is a derivative whose value is dependent on the price of a stock. However, derivatives can be dependent on almost any variable, from the price of hogs to the amount of snow falling at a certain ski resort.

There are two basic types of options. A ***call option*** gives the holder the right to buy the underlying asset by a certain date for a certain price. A ***put option*** gives the holder the right to sell the underlying assets by a certain date for a certain price. The price in the contract is known as the *exercise price* or *strike price*. The date in the contract is known as the *expiration date* or *maturity* [1].

The *style* or *family* of an option is a general term denoting the class into which the option falls, usually defined by the dates on which the option may be exercised. The vast majority of options are either ***European*** or ***American*** options. These options, together with others where the payoff is calculated similarly, are referred to as *vanilla options*. Options where the payoff is calculated differently are categorised as *exotic options*.

American options can be exercised at any time up to the expiration date. ***European options*** can be exercised only on the expiration date itself [1].

An option gives the holder the right to do something. The holder does not have to exercise the right. This is what distinguishes options from forwards and futures, where the holder is obligated to buy or sell the underlying asset.

4.1 Option positions

There are two sides to every option contract. On one side is the investor who has taken the long position (i.e. has bought the option). On the other side is the investor who has taken the short position (i.e. has sold or written the option). The writer of an option receives cash up front, but has potential liabilities later. The writer's profit or loss is the reserve of that for purchaser of the option. There are four types of option positions [1]:

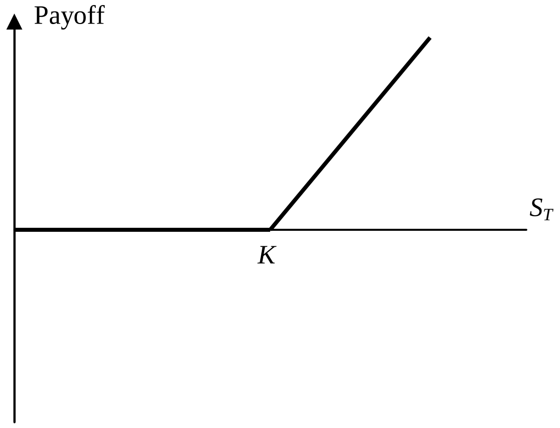
a) a long position in call option,
b) a long position in put option,
c) a short position in call option,
d) a short position in put option.

European options are generally easier to analyze than the American options, and some of the properties of an American option are frequently deduced from those of its European counterpart.

We will characterize European option positions in terms of the terminal value or payoff to the investor at maturity. The initial cost of the option is then not included in the calculation. If K is the strike price and S_T is the final price of underlying asset, the payoff from a long position in a European call option is [1]:

$$\max(S_T - K, 0) \tag{4.1}$$

This reflects the fact that the option will be exercised if $S_T > K$ and will not be exercised if $S_T \le K$.

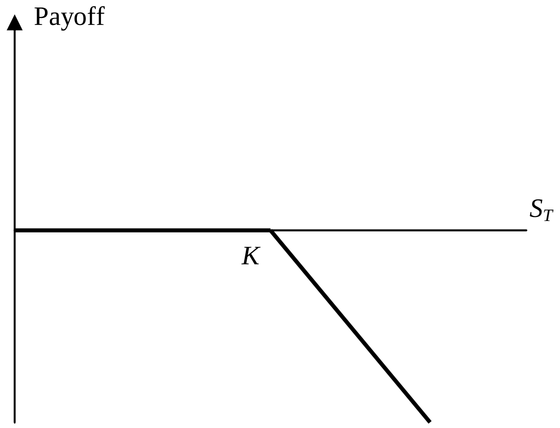


Source: Options, Futures and other Derivatives [1].

Picture 4.1: *Payoff from long position in European call option*

According to *HULL (2003)* the payoff to the holder of a short position in the European call option is

$-\max(S_T - K, 0) = \min(K - S_T, 0).$ (4.2)

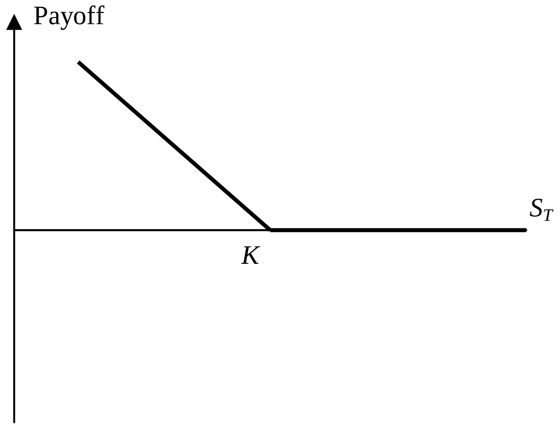


Source: Options, Futures and other Derivatives [1].

Picture 4.2: *Payoff from short position in European call option*

The payoff to the holder of a long position in a European put option is [1]:

$\max(K - S_T, 0)$. (4.3)

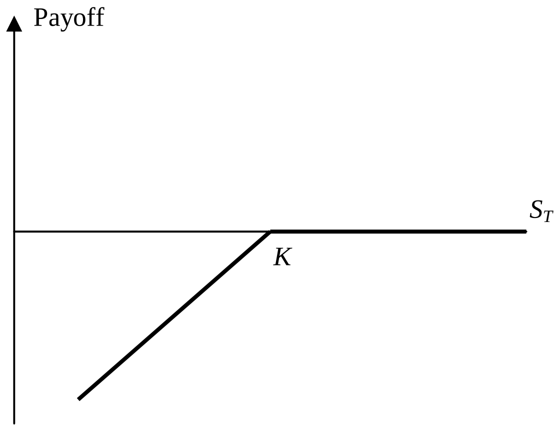


Source: Options, Futures and other Derivatives [1].

Picture 4.3: *Payoff from long position in European put option*

Finally, the payoff from a short position in a European put option is [1]:

$-\max(K - S_T, 0) = \min(S_T - K, 0)$ (4.4)



Source: Options, Futures and other Derivatives [1].

Picture 4.4: *Payoff from short position in European put option*

4.2 Terminology

Options are referred to as ***in the money***, ***at the money*** or ***out of the money***. "An in-the-money option would give the holder a positive cash flow if it were exercise immediately. Similarly, an at-the-money option would lead to zero cash flow if it were exercised immediately, and an out-of-the money option would lead to a negative cash flow if it were exercised immediately."[1]

HULL (2003) defined S as the stock price and K as the strike price. A call option is in the money when $S > K$, at the money when $S = K$, and out of the money when $S < K$. A put option is in the money when $S < K$, at the money when $S = K$ and out of the money when $S > K$. An option will be exercised only when it is in the money. In the absence of transaction costs, an in-the-money option will always be exercised on the expiration date if it has not been exercised previously.

The ***intrinsic value*** of an option is defined as "the maximum of zero and the value the option would have if it were exercised immediately"[1]. For a call option, the instrinsic value is therefore $\max(S - K, 0)$. For a put option it is $\max(K - S, 0)$. An in the money American option must be worth at least as much as its intrinsic value because the holder can realize a positive intrinsic value by exercising immediately. Often it is optimal for the holder of an in-the-money American option to wait rather than exercise immediately. The option is then said to have *time value.*

The ***time value*** of an option is "the part of the options value that derives the possibility of future favorable movements in the stock price"[1]. In general, the value of an option equals the intrinsic value of the option plus the time value of the option. The time value of an option is zero when [1]

a) the option has reached the maturity or
b) it is optimal to exercise the option immediately.

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[1] Hull, J. C. *Options*, *Futures and other Derivatives*. Fifth edition. New Jersey: PRENTICE HALL, 2003. ISBN 0130091448

There are six factors affecting the price of a stock option [1]:

- the current stock price S_0,
- the strike price K,
- the time to expiration T,
- the volatility of stock price σ,
- the risk free interest rate,
- the dividends expected during the life of the option.

Stock price and strike price

If a call option is exercised at some future time, the payoff will be the amount by which the stock price exceeds the strike price. Call options therefore are more valuable as the stock price increases and less valuable as the strike price increases. For a put option, the payoff on exercise is the amount by which the strike price exceeds the stock price. Put options therefore behave in opposite way from call options. They became less valuable as the stock price increases and more valuable as the strike price increases [3].

Time to expiration

Both put and call American options become more valuable as the time to expiration increases. We consider two options that differ only as far as the expiration date is concerned. The owner of the long life option has more exercise opportunities opened than the owner of the short life option. The long life option must therefore always be worth at least as the short life option [1].

Volatility of a stock price

The volatility of a stock price is a measure of how uncertain we are about future stock price movements. As the volatility increases, the possibility that the stock will do very well or very slenderly increases. For the owner of a stock, these two effects tend to offset each other. However the same does not applies for the owner of a call or put. The owner of a call option benefits from price increase but has limited downside risk in the case the price decreases because the owner can lose at most the price of the option. Similarly, the owner of a put benefits from price decreases, but has limited downside risk in the event of price increases. The values of both calls and puts therefore increase as volatility increases [8].

Risk-free interest rate

The risk-free interest rate affects the price of an option in a less straightforward way. When interest rates in the economy increase, the expected return required by investors from the stock has tendency to increase. Also, the present value of any future cash flow received by the holder of the option decreases. The combined impact of these two effects is that it decreases the value of put options and increases the value of call options [1].

Expected dividends

Dividends have the effect of reducing the stock price on the ex-dividend date. This is bad for the value of call option and good for the value of put option. The value of a call option has negative relation to the size of any expected dividends, and the value of a put option has positive relation to the size of any expected dividends [1].

4.3 Option styles

The *style* or *family* of an option is a general term denoting the class into which the option falls, usually defined by the dates on which the option may be exercised. The vast majority of options are either ***European*** or ***American*** options. These options, together with others where the payoff is calculated similarly, are referred to as *vanilla options* and they are described above. Options where the payoff is calculated differently are categorised as *exotic options*. Various option styles are decribed below.

4.3.1 Non-vanilla exercise rights

There exist unusual exercise styles in which the payoff value remains the same as a standard option (as in the classic American and European options above) but where the early exercise is different [8]:

- A *Bermudan option* is an option where the buyer has the right to exercise at a set (always discretely spaced) number of times. This is an intermediate between a European option – which allows to exercise at a single time, at expiry – and an American option, which allows to exercise at any time). The name of this style comes from Bermudas, which are situated between America and Europe.
- A *Canary option* is an option whose exercise style lies somewhere between European option and Bermudan option. Usually the holder can exercise the option at quarterly, but not before a set time period (usually one year) has elapsed.
- A *capped-style option* is an option, which is automatically exercised when the underlying security closes at a price making the option’s mark to market match the specified amount.
- A *compound option* is an option on another option, and as such presents the holder with two separate exercise dates and decisions.
- A *shout option* allows the holder effectively two exercise dates: during the life of the option he can (at any time) “shout” to the seller that he is locking-in the current price, and if this gives him a better deal than the pay-off at maturity he'll use the underlying price on the shout date rather than the price at maturity to calculate his final pay-off.

- A *swing option* gives the purchaser the right to exercise one and only one call or put on any one of a number of specified exercise dates (this latter aspect is Bermudan). Penalties are imposed on the buyer if the net volume purchased exceeds or falls below specified upper and lower limits. Allows the buyer to "swing" the price of the underlying asset. Primarily used in energy trading.

4.3.2 Exotic options with standard exercise styles

These options can be exercised either European style or American style. They differ from the plain vanilla option only in the calculation of their pay-off value [8]:

- A *cross option* (or a *composite option*) is an option on some underlying in one currency with a strike denominated in another currency. The pricing of such options needs to take into account FX volatility and the correlation between the exchange rate of the two currencies involved and the underlying stock price.
- A *quanto option* is a cross option in which the exchange rate is fixed at the outset of the trade, typically at 1.
- An *exchange option* is the right to exchange one asset for another.
- A *basket option* is an option on the weighted average of several underlyings.
- A *rainbow option* is a basket option where the weightings depend on the final performances of the components. A common special case is an option on the worst performing of several stocks.

4.3.3 Non-vanilla path dependent exotic options

The following exotic options are still options, but have payoffs calculated quite differently from those above. Although these instruments are far more unusual they can also vary in exercise style between European and American [8]:

- A *lookback option* is a path dependent option where the option owner has the right to buy (sell) the underlying instrument at its lowest (highest) price over some preceding period.
- An *Asian option* is an option where the payoff is not determined by the underlying price at maturity but by the average underlying price over some pre-set period of time.
- A *Russian option* is a lookback option which runs for perpetuity. That is, there is no end to the period into which the owner can look back.
- A *game option* or *Israeli option* is an option where the writer has the opportunity to cancel the option he has offered, but must pay the payoff at that point plus a penalty fee.
- A payoff of a *Parisian option* is dependent on the amount of time the option has spent above or below a strike price.
- A *barrier option* involves a mechanism where if a price is crossed by the underlying, the option either can be exercised or can no longer be exercised.
- A *binary option* (also known as a digital option) pays a fixed amount, or nothing at all, depending on the price of the underlying instrument at maturity.
- A *chooser option* gives the purchaser a fixed period of time to decide whether the derivative will be a vanilla call or put.

5 Option pricing models

Perhaps the most significant and revolutionary development in valuation is the acceptance, at least in some cases, that the value of an asset may not be greater than the present value of expected cash flows if the cashflows are contingent on the occurrence or non-occurrence of an event. This acceptance has largely come about because of the development of option pricing models. While these models were initially used to value traded options, there has been an attempt, in recent years, to extend the reach of these models into more traditional valuation. A lot of work has been done in the last twenty years in developing models that value options, and these option pricing models can be used to value any assets that have option-like features.

When we use option pricing models to value assets such as patents and undeveloped natural resource reserves, we are assuming that markets are sophisticated enough to recognize such options and to incorporate them into the market price. If the markets do not, we assume that they will eventually, with the payoff to using such models comes about when this occurs [1].

Option pricing models are variations on standard discounted cash flow models that adjust for management's ability to modify decisions as more information becomes available. Option models are particularly suitable for valuing strategic and operating flexibility such as opening and closing plants, abandoning operations, or natural resource exploration and development [6].

"An option can be valued as a function of the following variables - the current value, the variance in value of the underlying asset, the strike price, the time to expiration of the option and the riskless interest rate."[1] This was first established by Black and Scholes in 1972 and has been extended subsequently in numerous variants. While the Black-Scholes

[1] Hull, J. C. *Options, Futures and other Derivatives*. Fifth edition. New Jersey: PRENTICE HALL, 2003. ISBN 0130091448

option pricing model ignored dividends and assumed that options would not be exercised early, it can be modified to allow for both. Discrete-time variants, the Binomial and Trinomial option pricing models, have also been developed to price options.

According to *HULL (2003)* an asset can be valued as an option if the payoffs are a function of the value of an underlying asset. There are many assets that generally are not viewed as options but still have several option characteristics. We can view for instance equity as a call option on the value of the underlying firm, with the value of debt representing the strike price and term of the debt measuring the life of the option. A patent can be analyzed as a call option on a product, with the investment outlay needed to get the project going representing the strike price and the patent life being the time to expiration of the option.

The fundamental premise behind the use of option pricing models is that discounted cash flow models tend to understate the value of assets that provide payoffs that are contingent on the occurrence of an event.

5.1 Categorizing option pricing models

The first categorization of options is based on whether the underlying asset is a financial asset or a real asset. Most listed options are on financial assets such as stocks and bonds. In contrast, options can be on real assets such as commodities, real estate or investment projects. Such options are often called ***real options*** [1].

A second categorization according to *HULL (2003)* is based on whether the underlying asset is traded on not. Most financial assets are traded, while relatively few real assets are traded. Options on traded assets are generally easier to value and the inputs to the option models can be obtained from financial markets relatively easily. Options on non-traded assets are much more difficult to value since there are no market inputs available on the underlying asset.

5.2 Limitations of option pricing models

There are limitations in using option pricing models to value long term options on non-traded assets. The assumptions made about constant variance and dividend yields (which will be described further), which are not seriously contested for short term options, are much more difficult to defend when options have long lifetimes. When the underlying asset is not traded, the inputs for the value of the underlying asset and the variance in that value cannot be extracted from financial markets and have to be estimated. Thus the final values obtained from these applications of option pricing models have much more estimation error associated with them than the values obtained in their more standard applications (to value short term traded options) [8].

5.3 Theory of selected option pricing models

Option pricing theory has made a great progress since 1972, when Black and Scholes published their path-breaking paper providing a model for valuing dividend-protected European options. Black and Scholes used a “replicating portfolio” – a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued – to come up with their final formulation. While their derivation is mathematically complicated, there is a simpler binomial model for valuing options that draws on the same logic [8].

In the following section we will describe the theory of basic option pricing models such as Black-Scholes model, Binomial and Trinomial model.

5.3.1 Binomial model

In finance, the binomial options pricing model provides a generalisable numerical method for the valuation of options. The binomial model was first proposed by Cox, Ross and

Rubinstein in 1979. The model uses a discrete-time model of the varying price over time of the underlying financial instrument.

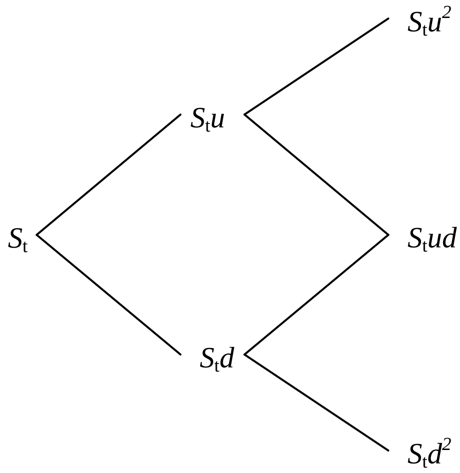
The binomial options pricing model approach is widely used as it is able to handle diverse conditions for which other models cannot easily be applied. The model is used to value American options which can be exercised at any point and Bermudan options which can be exercised at various points. The model is also mathematically relatively simple, and can be therefore implemented in a software environment.

Even though it is slower than the Black-Scholes model, it is considered more accurate, especially for longer-dated options and options on securities with dividend payments. Various versions of the binomial model are widely used in the options market.

The binomial pricing model uses a discrete-time framework to trace the evolution of the option's key underlying variable via a binomial tree, for a given number of steps between the valuation date and option expiration. Each node in the tree represents a possible price of the underlying asset, at a particular point in time. This price evolution is the basis for the option valuation [1].

The valuation process is iterative, starting at each final node, and then working backwards through the tree to the first node, where the calculated result is the value of the option [8].

The binomial option pricing model is based upon a simple formulation for the asset price process in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial is shown in picture 5.1.



Source: Damodaran online [8].

Picture 5.1 *General formulation for binomial price path*

In this picture, S_t is the current stock price; the price moves up to S_tu with probability p or down to S_td with probability $1 - p$ at any time period.

The assumptions used to derive the binomial equation are as follows [1]:

- It is discrete-time model.
- There exists the ideal capital market.
- The stationary probabilities of transition in single periods remain constant.
- From one state can come about only two possible conditions (increase or decrease of the price of the underlying asset).
- There are no transactions costs or taxes. All securities are perfectly divisible.
- The attitude of the investor is risk-neutral.
- There are no riskless arbitrage opportunities.
- Security trading is continuous.

- Stock prices follow the Markov process – stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant.
- The risk-free rate of interest r is constant and the same for lending and borrowing.

5.3.1.1 A one step binomial model

Firstly we will consider a one step binomial model which we will further develop into multiperiod binomial tree. To derive the formula of option pricing model we now consider a stock whose price is S_0 and an option on the stock whose current price is f. We suppose that the option last for time T and that during the life of the option the stock price can either move up from S_0 to a new level S_0u or down from S_0 to a new level S_0d, where $u > 1$ and $d < 1$. The proportional increase in the stock price when there is an up movement is $u - 1$; the proportional decrease when there is a down movement is $1 - d$. If the stock price moves up to S_0u, we suppose that the payoff from the option is f_u; if the stock price moves down to S_0d, we suppose that the payoff from the option is f_d [1].

HULL (2003) considers a portfolio which consists of a long position in Δ shares and a short position in one option. He calculates the value of Δ that makes the portfolio riskless. In the case there is an up movement in the stock price, the value of the portfolio at the end of the life of the option is

$$S_0 u\Delta - f_u . \tag{5.1}$$

In the case there is a down movement in the stock price, the value is

$$S_0 d\Delta - f_d . \tag{5.2}$$

The two are equal when

$$S_0 u\Delta - f_u = S_0 d\Delta - f_d \tag{5.3}$$

and then

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}. \tag{5.4}$$

In such case the portfolio is riskless and must earn the risk-free interest rate. Equation (5.4) shows that Δ is the ratio of the change in the option price to the change in the stock price as we move between the nodes. If we determine the risk-free interest rate as *r*, the present value of the portfolio will be then [1]

$$(S_0 u\Delta - f_u)e^{-rT}. \tag{5.5}$$

The cost to set up the portfolio is

$$S_0\Delta - f. \tag{5.6}$$

It follows that

$$S_0\Delta - f = (S_0 u\Delta - f_u)e^{-rT} \tag{5.7}$$

or

$$f = S_0 d - (S_0 u\Delta - f_u)e^{-rT}. \tag{5.8}$$

Substituting for Δ from equation (5.4) and simplifying reduces this equation to

$$f = e^{-rT}\left[pf_u + (1-p)f_d\right], \tag{5.9}$$

where

$$p = \frac{e^{rT} - d}{u - d}. \tag{5.10}$$

The variable *p* in the equation (5.9) is the probability of an up movement in the stock price. The variable *1 – p* is then the probability of a down movement and the expression

$$pf_u + (1-p)f_d \tag{5.11}$$

is the expected payoff from the option. With this interpretation of p, equation (5.9) then states that the value of the option today is its expected future value discounted at the risk-free rate [1].

The expected stock price $E(S_T)$, at time T, when the probability of an up movement is assumed to be p is given by [1]

$$E(S_T) = pS_0u + (1-p)S_0d \qquad (5.12)$$

or

$$E(S_T) = pS_0(u-d) + S_0d \ . \qquad (5.13)$$

If we substitute p from the equation (5.10), we obtain

$$E(S_T) = S_0e^{rT} \ , \qquad (5.14)$$

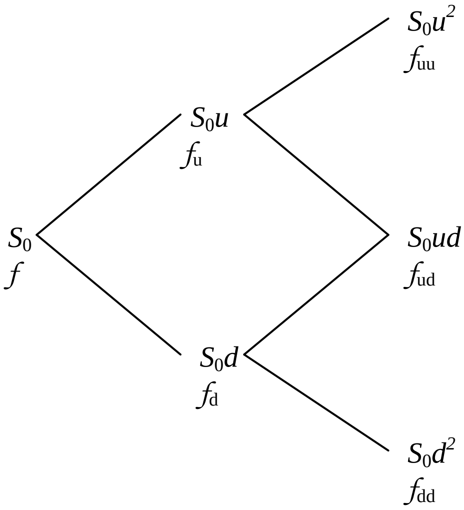
which shows that the stock price grows on average at the risk-free rate. Setting the probability of the up movement equal to p is therefore equivalent to assuming that the return on the stock equals the risk-free rate.

According to *HULL (2003)* in a risk-neutral world all individuals are indiferent to risk. In such world investors do not require any compensation for the risk, and the expected return on all securities is the risk-free interest rate. Equation (5.14) shows that we are assuming a risk neutral world when we set the probability of an up movement to p.

This is an important general principle in option pricing known as *risk-neutral valuation*. The principle states that we can assume the world is risk neutral when pricing an option. The price obtained from the calculation is correct in a risk-neutral world as well as in the real world.

5.3.1.2 Two step binomial tree

The stock price is initially S_0. During each time step, it either moves up to u times its initial value or moves down to d times its initial value.



Source: Options, Futures and other Derivatives [1].

Picture 5.2 *Stock and option prices in a general two-step tree*

The notation for the value of the option is shown on the tree in picture 5.2. We suppose that the risk-free interest rate is r and the length of the time step is δt years.

Repeated application of the equation (5.9) gives

$$f_u = e^{-r\delta t}\left[pf_{uu} + (1-p)\,f_{ud}\right], \tag{5.15}$$

$$f_d = e^{-r\delta t}\left[pf_{ud} + (1-p)\,f_{dd}\right], \tag{5.16}$$

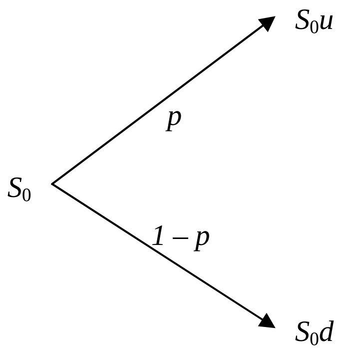
$$f = e^{-r\delta t}\left[pf_u + (1-p)\,f_d\right]. \tag{5.17}$$

Substituting from equations (5.15) and (5.16) into (5.17) we get

$$f = e^{-2r\delta t}\left[p^2 f_{uu} + 2p(1-p)\,f_{ud} + (1-p)^2\, f_{dd}\right]. \tag{5.18}$$

"The variables p^2, $2p(1-p)$, and $(1-p)^2$ are the probabilities that the upper, middle and lower final nodes will be reached. As we add more steps to the binomial tree, the risk neutral valuation principle continues to hold. The option price is equal to its expected payoff in a risk-neutral world, discounted at the risk-free rate."[1]

In practise, when we construct binomial tree to denote the movements in a stock price, we choose parameters u and d to match the volatility of the stock price. We suppose that the expected return on a stock is μ and its volatility is σ.



Source: Options, Futures and other Derivatives [1].

Picture 5.3: *Change in stock price in time δt in the risk neutral world*

The picture 5.3 shows stock price movements in the first step of a binomial tree. The step is of length δt. The stock price either can move up by a proportional amount u or moves down by proportional amount d. The probability of an up movement is assumed to be p.

The expected stock price at the end of the first step is $S_0e^{\mu\delta t}$. On the tree the expected stock price at this time is [1]:

$$pS_0u + (1 - p)S_0d \ . \tag{5.19}$$

[1] Hull, J. C. *Options*, *Futures and other Derivatives*. Fifth edition. New Jersey: PRENTICE HALL, 2003. ISBN 0130091448

In order to match the expected return on the stock with the tree's parameters, we must therefore have

$$pS_0u + (1-p)S_0d = S_0e^{\mu\delta t} \qquad (5.20)$$

or

$$p = \frac{e^{\mu\delta t} - d}{u-d} \qquad (5.21)$$

According to *HULL (2003)* the volatility of a stock price is defined so that $\sigma\sqrt{\delta t}$ is the standard deviation of the return on the stock price in a short period of time of length δt. Equivalently, the variance of the return is $\sigma^2\delta t$. On the tree in picture 5.3, the variance of the stock price return is

$$pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 . \qquad (5.22)$$

In order to match the stock price volatility with the tree's parameters, we must therefore have

$$pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 = \sigma^2\delta t . \qquad (5.23)$$

When we substitute p from equation (5.21) into equation (5.23), we get

$$e^{\mu\delta t}(u+d) - ud - e^{2\mu\delta t} = \sigma^2\delta t . \qquad (5.24)$$

When terms in δt^2 and higher powers of δt are ignored, one solution of this equation is

$$u = e^{\sigma\sqrt{\delta t}} , \qquad (5.25)$$

$$d = e^{-\sigma\sqrt{\delta t}} . \qquad (5.26)$$

The conditions which must hold is

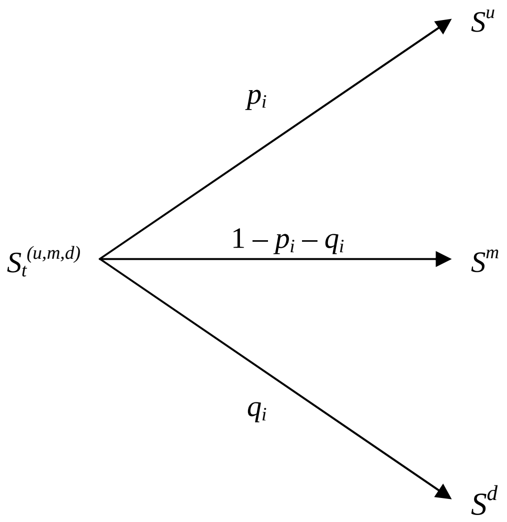
$$u \cdot d = 1 . \qquad (5.27)$$

The variable *e* is euler number and its value is 2,718.

5.3.2 Trinomial model

A trinomial tree with n levels is a set of nodes $S_t^{(u,m,d)}$ (representing the underlying price), where t = 1, ….,n is the level number, u, m and d are representing the possible movements of the price of underlying asset (u for the up movement, m is without movement and d for down movement) [8].

Being at node S_t, one can move to one of three nodes: (a) to the upper node with value S^u with probability p_i; (b) to the lower node with value S^d with probability q_i; and (c) to the middle node with value S^m with probability $1 - p_i - q_i$.



Source: Implied Trinomial Trees [6].

Picture 5.4 *Nodes in a trinomial tree*

A trinomial tree is a discrete representation of the evolution process of underlying prices. To apprehend and model the underlying price correctly, the tree has to meet following conditions [8]:

- it reproduces correctly the volatility,

- is risk-neutral,
- it uses transition probabilities from interval (0, 1).

To fulfill the risk-neutrality condition, the expected value of the underlying price $S_t^{(u,m,d)}$ in the following time period has to equal its known forward price [6]:

$$E(S_t^{(u,m,d)}) = p_i \cdot S_t^u + (1 - q_i - p_i) \cdot S_t^m + q_i \cdot S_t^d = F_0 = S_t^{(u,m,d)} e^{r\Delta t}, \tag{5.28}$$

where F_0 is the is the forward price, r denotes the continuous interest rate and Δt is the time step from t_n to t_{n+1}.

If we assume that σ is the stock price volatility during the time period, according to *CIZEK (2005)* we obtain second constraint on the node prices and transition probabilities:

$$p_i (S^u - F_0)^2 + q_i (S^d - F_0)^2 + (1 - p_i - q_i)(S^m - F_0)^2 = F_0^2 \sigma^2 \Delta t + o(\Delta t), \tag{5.29}$$

where $o(\Delta t)$ is the higher level than Δt.

We have now two constraints (5.28) and (5.29) for five unknown variables, which means that there is no unique implied trinomial tree. However all trees satysfying these constraints are equivalent in the sense that as the time spacing Δt tends to zero, all these trees converge to the same continuous process. A usual method for constructing a trinomial tree is to choose first randomly the prices of underlying assets and then to solve equations (5.28) and (5.29) for the transition probabilities p_i and q_i.

There exist few methods how to construct an initial state space. First we will focus on a construction of a constant-volatility trinomial tree, which is the base for implied trinomial tree.

5.3.2.1 Construction of a constant-volatility tree

We can start from a constant-volatility binomial tree and then combine two steps of this tree into a single step of a new trinomial tree. We can derive the following equations for the nodes of the trinomial tree:

$$S^u = S_{t+1}^u = S_t^{(u,m,d)} \cdot e^{\sigma\sqrt{2\Delta t}}, \tag{5.30}$$

$$S^m = S_{t+1}^m = S_t^{(u,m,d)}, \tag{5.31}$$

$$S^d = S_{t+1}^d = S_t^{(u,m,d)} \cdot e^{-\sigma\sqrt{2\Delta t}}, \tag{5.32}$$

where σ is a constant volatility.

We are also able to derive the up and down transition probabilities in the trinomial tree:

$$p_i = \left(\frac{e^{r\Delta t/2} - e^{-\sigma\sqrt{\Delta t/2}}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}} \right)^2, \tag{5.33}$$

$$q_i = \left(\frac{e^{\sigma\sqrt{\Delta t/2}} - e^{r\Delta t/2}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}} \right)^2, \tag{5.34}$$

or

$$p_i = \sqrt{\frac{\Delta t}{12\sigma^2}} \left(r - \frac{\sigma^2}{2} \right) + \frac{1}{6}, \tag{5.35}$$

$$q_i = -\sqrt{\frac{\Delta t}{12\sigma^2}} \left(r - \frac{\sigma^2}{2} \right) + \frac{1}{6}, \tag{5.36}$$

$$1 - p_i - q_i = 2/3. \tag{5.37}$$

5.3.2.2 Construction of a trinomial tree with changing volatility

If the volatility varies significantly with strike or time to maturity, we should choose a state space which reflects such properties.

"Let $C(K, t_{n+1})$ and $P(K, t_{n+1})$ represent today's price of a standard European call and put option, respectively, struck at K and expiring at t_{n+1}. The values of these options given by the trinomial tree are the discounted expectations of the pay-off functions: $\max(S_j - K, 0) =$

$(S_j - K)^+$ for the call option and $\max(K - S_j, 0)$ for the put option at the node $t + 1$, (u, m, d)."[1]

The expectation is taken with respect to the transition probabilities [6]:

$$C(K, t+1) = e^{-r\Delta t} \sum_j \{p_j \lambda_{n,j} + (1 - p_{j-1} - q_{j-1})\lambda_{n,j-1} + q_{j-2}\lambda_{n,j-2}\}(S_j - K)^+, \tag{5.38}$$

$$P(K, t+1) = e^{-r\Delta t} \sum_j \{p_j \lambda_{n,j} + (1 - p_{j-1} - q_{j-1})\lambda_{n,j-1} + q_{j-2}\lambda_{n,j-2}\}(K - S_j)^+ . \tag{5.39}$$

If we set the strike price K to S^m, we can set the transition probabilities p_i and q_i for all nodes above the central node from equation:

$$p_i = \frac{e^{r\Delta t} C(S_{t+1}^m) - \sum_{j=1}^{i-1} \lambda_{n+1,j} (F_j - S^m)}{\lambda_{n+1,j} (S^{(u,m,d)} - S^m)}, \tag{5.40}$$

$$q_i = \frac{F_i - p_i (S^{(u,m,d)} - S^m) - S^m}{S^d - S^m} . \tag{5.41}$$

Similarly we can express the transition probabilities for all nodes below (and including) the center node [6]:

$$q_i = \frac{e^{r\Delta t} P(S_{t+1}^m) - \sum_{j=m}^{2n-1} \lambda_{n+1,j} (S^m - F_j)}{\lambda_{n+1,j} (S^m - S^d)}, \tag{5.42}$$

$$p_i = \frac{F_i - q_i (S^d - S^m) - S^m}{S^{(u,m,d)} - S^m} . \tag{5.43}$$

The variable $\lambda_{n,i}$ is the known Arrow-Debreu price at node (n, i). The Arrow-Debreu prices for a trinomial tree can be obtained by the following iterative formulas [1]:

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[1] CIZEK, P., KOMORAD, K. *Implied Trinomial Trees*. Berlin: Humboldt University, 2005.

$$\lambda_{1,1} = 1, \quad (5.44)$$

$$\lambda_{n+1,1} = e^{-r\Delta t}\lambda_{n,1}p_1, \quad (5.45)$$

$$\lambda_{n+1,2} = e^{-r\Delta t}\{\lambda_{n,1}(1-p_1-q_1)+\lambda_{n,2}p_2\}, \quad (5.46)$$

$$\lambda_{n+1,i+1} = e^{-r\Delta t}\{\lambda_{n,i-1}q_{i-1}+\lambda_{n,i}(1-p_i-q_i)+\lambda_{n,i+1}p_{i+1}\}, \quad (5.47)$$

$$\lambda_{n+1,2n} = e^{-r\Delta t}\{\lambda_{n,2n-1}(1-p_{2n-1}-q_{2n-1})+\lambda_{n,2n-2}q_{2n-2}\}, \quad (5.48)$$

$$\lambda_{n+1,2n+1} = e^{-r\Delta t}\lambda_{n,2n-1}q_{2n-1}. \quad (5.49)$$

5.3.3 Black-Scholes model

In the early 1970s, Fisher Black, Myron Scholes and Robert Merton made a major breakthrough in the pricing of stock options. This involved the development of what has become known as the Black-Scholes model.

The model has had a huge influence on the way that the traders price and hedge options. It has also been pivotal to the growth and success of financial engineering in the 1980s and 1990s.

The Black-Scholes-Merton differential equation is an equation that must be satisfied by the price of any derivative dependent on a non-divident-paying stock. The assumptions used to derive the Black-Scholes differential equation are as follows [1]:

- The stock price follows the geometric Brownian motion with logaritmic prices and with μ and σ constant.
- There exists the ideal capital market.
- The short selling of securities with full use of proceeds is permitted.
- There are no transactions costs or taxes. All securities are perfectly divisible.
- There are no dividends during the life of the derivative.
- There are no riskless arbitrage opportunities.
- Security trading is continuous.
- The risk-free rate of interest r is constant and the same for all maturities.

When the price process is continuous, which means that price changes becomes smaller as time periods get shorter, the binomial model for pricing options converges on the Black-Scholes model. The model allows us to estimate the value of any option using a small number of inputs.

It is based on the idea of creating a portfolio of the underlying asset and the riskless asset with the same cashflows and hence the same cost as the option being valued. The value of a call option in the Black-Scholes model can be written as a function of the five variables [1]:

S_0 = current value of the underlying asset,
K = strike price of the option,
T = life to expiration of the option,
r = riskless interest rate corresponding to the life of the option,
σ = variance in the price of the underlying asset.

The Black-Scholes formulas for the prices at time zero of a European call option on a non-dividend-paying stock and a European put option on a non-dividend-paying stock are

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{5.50}$$

and

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1), \tag{5.51}$$

where

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}, \tag{5.52}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}. \tag{5.53}$$

The variables c and p are the European call and put price. Note that e^{-rt} is the present value factor and reflects the fact that the exercise price on the call option does not have to be paid until expiration. The functions $N(d_1)$ and $N(d_2)$ are the probabilities, estimated by using a cumulative standardized normal distribution and the values of d_1 and d_2 obtained for an option [1].

The value of a put option can be derived from the value of a call option with the same strike price and the same expiration date. Two options are considered [8]:

- one European call option plus an amount of cash equal to Ke^{-rt},
- one European put option plus one share.

Both are worth

$$\max(S_T, K) \tag{5.54}$$

at expiration of the options. Because the options are European, they cannot be exercised prior to the expiration date. The options must therefore have identical values today. This means that

$$c + Ke^{-rT} = p + S_0 \,. \tag{5.55}$$

This relationship is known as ***put-call parity***. It shows that the value of a European call with a certain exercise price and exercise date can be deduced from the value of a European put with the same exercise price and exercise date, and vice versa.

Because the European price equals the American price when there are no dividends, equation (5.50) also gives the value of an American call option on a non-dividend-paying stock. Unfortunately no exact analytic formula for the value of an American put option on a non-dividend-paying stock has been produced.

5.3.3.1 Model limitations and fixes

The Black-Scholes model was designed to value options that can be exercised only at maturity and on underlying assets that do not pay dividends. In addition, options are valued based upon the assumption that option exercise does not affect the value of the underlying asset. In practice, assets do pay dividends, options sometimes get exercised early and exercising an option can affect the value of the underlying asset. Therefore there exist adjustments. While they are not perfect, adjustments provide partial corrections to the Black-Scholes model [8].

1. *Dividends*

The payment of a dividend reduces the stock price. On the ex-dividend day, the stock price generally declines. Consequently, call options will become less valuable and put options more valuable as expected dividend payments increase. There are two ways of dealing with dividends in the Black-Scholes model [8]:

- **Short-term options:** One approach how to deal with dividends is to estimate the present value of expected dividends that will be paid by the underlying asset during the life of the option and substract it from the current value of the asset to use as S_0 in the model.

  Modified stock price = current stock price – present value of expected dividends during the life of the option (5.56)

- **Long-term options:** As the option life becomes longer, it is better to choose an alternative approach. If the dividend yield (y = dividends/current value of the asset) on the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

  $$c = S_0 e^{-yT} N(d_1) - K e^{-rt} N(d_2)\,, \tag{5.57}$$

where

$$d_1 = \frac{\ln(S_0 / K) + (r - y + \sigma^2 / 2)T}{\sigma\sqrt{T}}, \tag{5.58}$$

$$d_2 = d_1 - \sigma\sqrt{T}. \tag{5.59}$$

The adjustments have two effects. First the value of the asset is discounted back to the present at the dividend yield to take into account the expected drop in asset value resulting from dividend payments. Second, the interest rate is offset by the dividend yield to reflect the lower carrying cost from holding the asset in the replicating portfolio.

2. *Early exercise*

The Black-Scholes model was designed to value options that can be exercised only at expiration. Options with this characteristic are called **European options**. In contrast, most options that we encounter in practice can be exercised any time until expiration. These options are called **American options**. The possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. In general, though, with traded options, it is almost always better to sell the option to someone else rather than exercise early, since options have a time premium, i.e., they sell for more than their exercise value [8].

3. *The impact of exercise on the value of the underlying asset*

The Black-Scholes model is based upon the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for some types of options. For instance, the exercise of warrants increases the number of shares outstanding and brings fresh cash into the firm, both of which will affect the stock price. The expected negative impact of exercise will decrease the value of warrants compared to otherwise similar call options. The adjustment for dilution in Black-Scholes to the stock price is quite simple – the stock price is adjusted for the expected dilution from the exercise of the options [8].

5.4 Real options

The option pricing methodology is exercised in many areas of financial management and decision making because its advantage is elimination of some imperfections and assumptions on which the traditional models and approaches are based. Such methods assume the passive approach, which means that the strategy the company has set will be held. However the majority of projects have long-term character and thus the expected values will be similar to the real values only in exceptional cases [2].

The option methodology perceives the investment process as an active approach which assumes the active intervention of the company's management and which can be perceived as process of management of the real options portfolio with the objective of maximalization of its value or possibly minimalization of its losses [2].

The option pricing methodology enables to evaluate the investment projects. It quantifies and includes into the management's decision making also the value of the options, which in the case of investment project are related to the possibilities to make changes during its lifetime. This is important when the market conditions develop differently from the expectations and when the new possibilities and opportunities for the company emerge [2].

"A real option is the right, but not the obligation, to undertake some business decision, typically the option to make a capital investment. This kind of option is not a derivative instrument, but an actual tangible option (in the sense of choice) that a business may gain by undertaking endeavors."[1] It can be represented for example by investing in a project or property, by the real option of expanding, downsizing or abandoning other projects in the future. Other examples can be opportunities for research and development, mergers and acquisitions, licensing etc. "They are called real options because they pertain to physical or tangible assets, such as equipment, rather than financial instruments."[1]

5.4.1 Basic factors affecting the price of a real option

The principal assumption for the application of real option pricing methodology is the possibility to define the basic variables that affect the price of a real option. As with financial options, the value of a real option depends on five parameters [4]:

- The market value of the underlying asset on which the option is contingent,
- the exercise price of the option,
- the time remaining until the maturity of the option,
- the volatility of the underlying asset,
- the risk-free rate of interest.

These are clearly defined for financial options, but require better understanding for real options.

Underlying asset

In the case of real options, the underlying asset is the cash flow of the project in time *t*. The higher is the value of the underlying asset, the higher is the value of call option and viceversa for a put option.

[1] Copeland, T., Antikarov, V. *Real Options: Practitioner´s Guide*. London, New York: TEXERE, 2000. ISBN 1-58779-028-8.

Exercise price of the option

The exercise price of the real option is the value of investment costs which have to be spent in the case of exercise of call option. The exercise price of a put option is represented by the saved investment costs.

Time remaining until the maturity of the option

The time remaining until the maturity of the option is the time period during which it is possible to exercise the option. Usually it is assumed that the option can be exercised anytime during the lifetime of the project (that concerns the American option); if the given right is possible to exercise only in particular year it concerns the European option.

The volatility of the underlying asset

The value of the option and hence the value of the whole project is the higher, the higher is the risk of underlying asset expressed by its volatility. This attribute holds for put as well as for call options because it increases the probability of their exercise. The risk parameter of the project and its impact on the value of the project represents the key difference in comparison with traditional sales comparison approaches. Whilst for these approaches the increasing risk decreases the value of the project, in the case of real option pricing methodology it holds viceversa.

The risk-free rate

An increase in the risk-free rate will increase option value since it will increase the time value of money advantage in deffering the investment cost.

5.4.2 Comparison of real options and stock options

An important difference between financial and real options is that the management can affect the value of the underlying risky asset (a physical project under its control) while financial options are side bets owned by third parties that cannot affect the outcome of the underlying asset.

Stock option	***Real option***
▪ Price of the underlying asset	▪ Present value of future cash flows
▪ Exercise price	▪ Investment cost
▪ Time remaining until the maturity of the option	▪ Lifetime of the project
▪ Price volatility of the underlying asset	▪ Cash flow volatility of the project
▪ Risk-free interest rate	▪ Risk-free interest rate

5.4.3 Classification of real options

To identify potential operating flexibility and strategic factors which the real options provide to the management, we can classify real options into following categories [1]:

- ***Option to expand the project***

This is the option to make further investments and increase the output if conditions are favorable. Management might choose to build capacity in excess of the expected level of output so that it can manufacture at a higher rate if the product is more successful than was anticipated. The expansion option gives to management the right but not the obligation, to make additional follow-on investment if project conditions turn out to be favorable [7].

It is an American call option on the value of additional capacity. The strike price of the call option is the cost of creating this additional capacity discounted to the time of option exercise. The strike price often depends on the initial investment. If management initially chooses to build capacity in excess of the expected level of output, the strike price can be relatively small [1].

- ***Option to contract the project***

This is the option to reduce the scale of a project's operation. It is an American put option on the value of the lost capacity. The strike price is the present value of the future expenditures saved as seen at the time of exercise of the option [1].

- ***Option to abandon the project***

This is an option to sell or close down the project. It is formally equivalent to an American put option on a stock.

If the bad outcome occurs at the end of the first period, the decision maker may abandon the project and realize the expected liquidation value. Then the exercise price of the put is the expected liquidation (or resale) value of the project. When the present value of the asset falls below the liquidation value, the abandoning (or selling) the project is equivalent to exercising the put. Because the liquidation value of the project sets a lower bound on the value of the project, the option to liquidate is valuable. A project that can be liquidated is worth more than the same project without the possibility of abandonment [2].

- ***Option to defer the project***

The option to defer an investment to develop a property is formally equivalent to an American call option on the stock.

For example the owner of a lease on an undeveloped oil reserve has the right to acquire a developed reserve by paying a lease-on-development cost. The owner can defer the development process until oil prices rise. Or differently, the managerial option implicit in holding an undeveloped reserve is a deferral option. The expected development cost is the exercise price of the call. *McKINSEY (2000)* has said that the net production revenue less depletion of the developed reserve is the opportunity cost incurred by deferring the investment. If this opportunity cost is too high, the decision maker may want to exercise the option (that is, develop the reserve) before its relinquishment date.

- ***Option to extend***

Sometimes it is possible to extend the life of an asset by paying a fixed amount. This is a European call option on the asset's future value [1].

- ***Switching options***

The option to switch project operations is a portfolio of options that consists of both calls and puts. Restarting operations when a project is shut down is an American call option. Shutting down operations when unfavorable conditions arise is an American put option.

The cost of restarting (or shutting down) operations may be thought of as the exercise price of the call (or put). A project whose operation can be turned on and off (or switched between two distinct locations, etc.) is worth more than the same project without the flexibility to switch. "A flexible manufacturing system with the ability to produce two products is a an example of this type of option, as is peak-load power generation and the ability to exit and reenter an industry."[1]

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[1] McKinsey & Company Inc, Copeland, T., Koller, T., Murrin, J. *Valuation – Measuring and Managing the Value of Companies*. Third edition. New York: JOHN WILEY & SONS, INC., 2000. ISBN 0-471-36190-9

- ***Compound options***

These are the options on options. Good example of such options are phased investments. We can consider a factory that can be built as a sequence of real options, each contingent on those that precede it. The project can be continued at each stage by investing a new amount of money (an exercise price). Alternatively, it might be abandoned for whatever it can fetch. Other examples are research and development programs, new product launches, exploration and development of oil and gas fields, and an acquisition program where the first investment is thought of as a platform for later acquisitions [2].

Another example can be the equity of indebted company as a call option on the value of the company, whose exercise price is the nominal value of the company's debt and time to expiration is identical with the time of maturity. Call option on the equity, where the equity is the underlying asset, is then an option on option. These compound options are so called simultaneous compound options. The option on the value of indebted company and the option on equity have the same time to expiration and they exist simultaneously [8].

5.5 Equity in highly levered distressed firms

In most publicly traded firms, equity has two features. The first is that the equity investors run the company and they can choose to liquidate its assets and pay off other claim holders at any time. The second feature is that the liability of equity investors in some private firms and almost all publicly traded firms is restricted to their equity investments in these firms. Such combination of the option to liquidate and limited liability allows equity to have characteristics of a call option [8].

5.5.1 The payoff on equity as an option

The equity in a firm is a residual claim, it means that equity holders lay claim to all cash flows left after other financial claimholders (debt, preferred stock, etc.) have been satisfied. If a firm is liquidated, the same principle applies; equity investors receive the cash that is left in the firm after all outstanding debt and other financial claims have been paid off. With limited liability, if the value of the firm is less than the value of the outstanding debt, equity investors cannot lose more than their investment in the firm. The payoff to equity investors on liquidation can therefore be written as [8]:

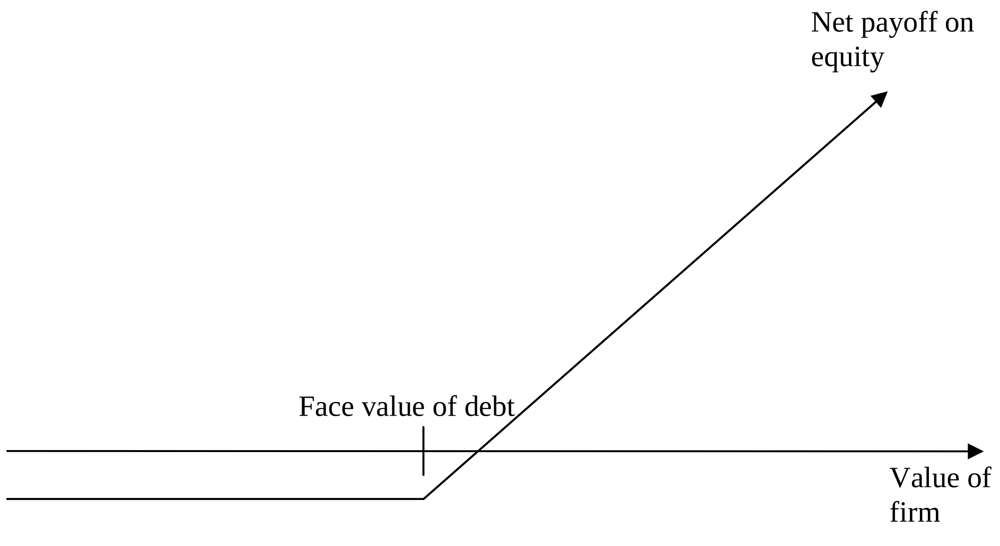
$$\text{Payoff to equity on liquidation} = V - D, \text{ if } V > D, \quad (5.60)$$

$$= 0, \text{ if } V \leq D, \quad (5.61)$$

where

V = liquidation value of the firm,
D = face value of the outstanding debt and other external claims.

Equity can be therefore viewed as a call option on the firm, where exercising the option requires that the firm is liquidated and the face value of the debt (which corresponds to the exercise price) is paid off. The firm is the underlying asset and the option expires when the debt comes due. The payoffs are shown in picture 5.5.



Source: Damodaran online [8].

Picture 5.5: *Valuing equity as an option*

5.5.2 Estimating the value of equity as an option

Option pricing model to value equity is based on following assumptions [8]:

- There are only two claimholders in the firm – debt and equity.
- There is only one issue of debt outstanding and it can be retired at face value.
- The debt has a zero coupon with time to expiration *T* and nominal value of the debt *D*.
- The value of the firm and the variance in that value can be estimated.

Every assumption stated above has a reason to exist. The problem becomes more simple when we restrict the claimholders to just debt and equity. Introducing other claimholders such as preferred stock would make it more difficult to get the result. When we assume only one zero-coupon debt issue which can be retired at face value at any time before the maturity, we align the features of the debt more closely to the features of the strike price on a standard option. If the debt is a coupon debt, or more than one debt issue is outstandin, the equity investors can be forced to exercise (liquidate the firm) at these earlier coupon dates if they do not have the cash flows to meet their coupon obligations [8].

Finally, knowing the value of the firm and the variance in that value makes the option pricing possible. If the debt of a firm is not publicly traded, the option pricing model can provide an estimate of the value for the equity in the firm. When the debt is publicly traded, there exists a possibility that the bond are not correctly valued and the option pricing theory can verify whether the values of the debt and equity are right [5].

5.5.3 Inputs for valuing equity as an option

Since most of companies does not meet the criteria stated above (such as having only one zero-coupon bond outstanding), few compromises have to be done to use the model in valuation.

5.5.3.1 Value of the firm

According to *DAMODARAN (2007)*, value of the firm can be obtained in one of three ways. The first approach is that we cumulate the market values of outstanding debt and equity, assuming that all debt and equity are traded, to obtain the value of company. The option pricing model then reallocates the firm value between debt and equity.

In the second approach, we estimate the market values of the assets of the company by discounting expected cash flows at the cost of capital. The value of the firm obtained from option pricing model should be the value obtained on liquidation.

The third approach consists in estimating a multiple of revenues by looking at healthy firms in the same business and applying this multiple to the revenues of the firm you are valuing. We assume that a potential buyer will pay such value in the case of liquidation.

5.5.3.2 Variance in firm value

The variance in the firm value can be obtained directly if both stocks and bonds in the firm are traded. We define σ_E^2 as the variance in the stock price and σ_D^2 as the variance in the bond price, w_E as the market-value weight of equity and w_D as the market-value weight of debt, we can write the variance in firm value as [8]

$$\sigma_{FIRM}^2 = w_E^2 \cdot \sigma_E^2 + w_D^2 \cdot \sigma_D^2 + 2 \cdot w_W \cdot w_D \cdot \sigma_E \cdot \sigma_D \cdot \rho_{ED}, \tag{5.62}$$

where ρ_{ED} is the correlation between the stock and bond prices.

In the case the bonds of the company are not traded, we can use the variance of similarly rated bonds as the estimate of σ_D^2 and the correlation between similarly rated bonds and the company's stock as the estimate of σ_E^2.

5.5.3.3 Trend analysis of the past development

Trend analysis is can be used always when we need to determine the basic trend of development of analysed time series at least approximately. The trend line can be expressed as [7]

$$T_t = a + bt\,, \tag{5.63}$$

where *a* and *b* are the unknown parameters of trend line and *t* = 1,2, …, n is the time variable. Parameter *a* states the value of the trend in time *t* = 0, and parameter *b* expresses the change of trend when the value of time variable increases by 1 unit.

Parameters *a* and *b* are calculated as follows [7]:

$$a = \bar{y} - b\bar{t}\,, \tag{5.64}$$

$$b = \frac{\sum t y_t - \bar{t}\sum y_t}{\sum t^2 - n\bar{t}^2}\,. \tag{5.65}$$

In association with the trend analysis is often mentioned the autocorrelation coefficient, which is the analysis of dependence between adjacent members of one time series. The degree of closeness of this dependence is the autocorrelation koeficient of first order r_1 [7]:

$$r_1 = \frac{(n-1)\sum_{t=1}^{n-1} y_t y_{t+1} - \sum_{t=1}^{n-1} y_t \sum_{t=1}^{n-1} y_{t+1}}{\sqrt{\left[(n-1)\sum_{t=1}^{n-1} y_t^2 - \left(\sum_{t=1}^{n-1} y_t\right)^2\right]\left[(n-1)\sum_{t=1}^{n-1} y_{t+1}^2 - \left(\sum_{t=1}^{n-1} y_{t+1}\right)^2\right]}}\,. \tag{5.66}$$

5.5.3.4 Maturity of the debt

Most firms have more than one debt issue on their books and much of the debt comes with coupons. As one of the assumptions of the option pricing model allows for only one input for the time to expiration, we have to convert these multiple bonds issues and coupon payments into one equivalent zero-coupon bond.

The solution of this problem, which considers both the coupon payments and the maturity of the bonds, is the estimation of the duration of each debt issue and calculating a face-value-weighted average of the durations of the different issues. Such value-weighted duration is then used as a measure of the time to expiration of the option.

According to *HULL (2003)*, the duration of a bond is a measure of how long on average the holder of the bond has to wait before receiving cash payments. A zero-coupon bond that matures in *n* years has a duration of *n* years. We suppose that a bond provides the holder with cash flows c_i at time t_i ($1 \le i \le n$). The price B, and yield y are related by

$$B = \sum_{i=1}^{n} c_i e^{-yt_i} \ . \tag{5.67}$$

The duration D of the bond is defined as

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right]. \tag{5.68}$$

The term in square brackets is the ratio of the present value of the cash flow at time t_i to the bond price. The bond price is the present value of all payments. The duration is therefore a weighted average of the times when payments are made, with the weight applied to time t_i being equal to the proportion of the bond's total present value provided by the cash flow at time t_i. The sum of the weights is 1 [1].

5.5.3.5 Face value of the debt

In the case the company has multiple debt issues outstanding, there are three posibilities what to use as a face value of the debt [8]:

- We can add up the principal due on all of the debt of the firm and consider it to be the face value of the hypothetical zero coupon bond that we assume the company has issued.

- We can add the expected interest and coupon payments that will come due on the debt to the principal payments to come up with accumulated face value of the debt. This is the simplest approach of dealing with intermediate interest payments coming due.
- We can consider only the principal due on the debt as the face value of the debt and the interest payments each year, specified as a percent of firm value, can take the place of the dividend yield in the option pricing model.

6 Valuation of the company as a real option

For the practical part of the Master Thesis has been chosen the company operating in armament industry, Ceska Zbrojovka a.s. The valuation will be performed by application of trinomial discrete-time model.

The company will be valued as an American call option on the value of the underlying firm, where the underlying asset is considered the market value of assets and the value of debt representing the strike price.

In the following part there will be described the armament industry and the situation in the Czech Republic, then there will be presented basic data about the valued company and finally there will be described the whole procedure and process of the valuation.

In the valuation will be assumed that the chosen factors will be developing according to the trinomial tree in time period 2006 – 2011, and in the second stage will be assumed that the values remain constant and unchanged as in 2011. It is assumed that the company will exist infinitely long time. As a moment of valuation has been chosen the year 2006.

6.1 Analysis of armament industry

The armament industry has traditionally very close connotations with the defense and therefore with the questions of state sovereignty. Thus it is logic that every advancement in armament techniques provokes a political reaction.

Even though there have been established four freedoms within the European Union, the products and services of armament industry have been excluded from this regime. It mean that EU member states keep the discretion in determining of import tariffs, they burden the export and import with administrative duties and there does not exist any business policy.

With the end of bipolar system the costs, connected with the purchase and development of military weapons systems, have rapidly increased. This was accompanied by decrease of public expenses on defense, because the Europeans arrived at conviction that there is no more need to maintain high armament growth rate, as the Warsaw Treaty and Soviet Union have fell apart. Together with it, the restructuring of armies of European countries aimed to professionalization. In the case of post communist countries of East and Central Europe the above mentioned implications have been accompanied by the need of rearmament, which was related to conditions negotiated before the admission to NATO at the end of 1990s.

The European armament industry was not able to produce sofisticated weapon systems requiring huge investments in research and development. Thus the ability of European production to compete with the transatlantic producers has decreased.

The key player on European armament market remains the sovereign member state. This is in virtue of historic tradition, which considers the production of military technologies as a strategic sector which influences the striking power of the state. Therefore the governments keep high degree of authority in this sector.

State is practically the only customer which influences the demand for products depending on strategic planning. Thus is defined the extent of armament market of that country.

State is also significant investor in military research and development, whereby it helps to increase the technological know-how and thus to increase the competitivness of whole industry.

The third function of state is its regulatory character of its competencies. The responsible authorities of EU member states impose exercise licences on exports or they approve the conditions of competition of tenders on purchase of military goods.

States tend to rely on their own production rather than on imports from other countries which can be their potential enemies. Other specificity is the latency of works on armament programme which can higly influence the technological dominance of belligerent state. Therefore the states guard rigorously the access to information which could lead to revelation of technical details of weapon components.

Armament production is nowadays very complex and costly process with many impacts on other areas. While the size of production is often limited by the factual demand, the risk of commercial failure is rising steadily which logically makes space for state subsidies in cases where market mechanisms cannot be applied. Current armament systems include many items that could be used in civil productions. Long life cycle of these systems is around fifty years in standard. Producer must handle the investment risks throughout this period. However the product price must be strictly connected with its quality.

The purchases of complete military products and materials (off-the-shelf) are usually realized through the offset conditions. This provides an opportunity of return of investment for the purchasing country which can exceed 100% of the price. Offsets are either direct (sub suppliers are from the purchasing country or the transfer of production technology is arranged) or indirect (investments to other than military sector).

Legal ground for an exclusion of armament market from the process of European integration is Article 296 of The European Economic Community Treaty (1957) where the Treaties of Rome were arranged and EEC and Euroatom established. According to this Article, any member state cannot be forced to provide information, which could endanger its safety.

It was strictly defined that armament markets will be regulated by the member states from now on. However in 90´s The European Commission brought forward a few suggestions for the armament cooperation on the EU level. The regulation of The European Commission specify the interpretation of the 296 article in such way that the defence industry falls within European legislation cognizance in the area of competition of tenders in cases of civil goods purchases and goods which does not have specific military purpose even if the purchase is conducted by the member's state Ministry of defence.

The second area of problems (except for 296. article)is brought by the difference between the national legislations. Here works the different systems in the ways of public presentation and in the competition of tenders' specifications. The quasidiscrimination of the abroad companies could be seen here. The countries usually define the technical conditions of contract so that only home private subjects can realize them.

The budgetary restrictions, insufficient size of national markets and transformation of many EU member states' armies cause the neglect of research and development of higher generations of military equipment. Further due to ineffective allocation of resources the additional costs are increased and delivery terms are prolonged as well.

Generally the European armament industry loses the ability to compete on world market. The European Union spends 10 billions EUR on research and development every year, which creates 13 per cent of total EU GDP, whereas the United States detach from its budget 15 per cent of GDP on research purposes.

The armament industry has a long tradition in the Czech Republic – in years 1934 and 1935 was then Czechoslovakia the world's biggest exporter of military material. Even in 1980s the Czechoslovakia get around the 7th place on worldwide list of gun exporters and exports of military material represented up to 10 per cent of whole Czechoslovak export.

After the year 1990 the environment has changed, in 1992 the government stopped the support to armament orders and as a result of changes political situation came to restraint of armament production and to its restructuring. The production has decreased by nine tenth in comparison with the situation in half of 1980s. Large number of companies has left the industry for reason of problematic privatization.

The break up of the Warsaw Pact meant a hard swipe for a domestic armament industry. Hovewer in last years the sector begins to break through. The partial cause of this success is the change of market requirements. Instead of big armoured cars is perspective the sphere of special electronics or chemistry.

Last year, export of guns from the Czech Republic has increased by approximately 6 per cent on 2,6 billion CZK. However the guns are concerned in total export only by 0,12 per cent. Czech companies have exported mainly products embedded in group including army tanks, armoured cars and their special components, which created 34 per cent of total export. Aeronautical technologies and export of services (such as technical help and training) have also succeeded.

Half of the weapon exports are heading to Europe, to South America go 15 per cent and the rest is proportionally distributed over the world.

To the contrary, the import of weapons has rapidly decreased on 2,63 billion CZK. For the first time after many years, the balance between export and import has been reached.

Arms dealers and producers have had to cope with heavy decrease of commissions after the end of Cold War. "Fortunately for them, new phenomenon has arised. The risks arising from the world terorism, accompanied by suicidal bomb attacks, fears from use of chemical and biological weapons or computer attacks, create new opportunities for armament industry.

The situation between the "rich North and poor South" accompanied by the fanatism and fight of the Great Powers for natural resources will be continually worsening. Besides the increased interest in solution of such risks, the differences between armament and non-military production will be diminishing and the role of dual technologies (those which are applicable on civil and military purposes) will be strenghtening. Companies, which will be able to addapt to such conditions, will be able to compete in future.

6.2 Analysis of the company Ceska zbrojovka a.s.

Ceska zbrojovka a.s. is a company operating in the precision mechanical engineering field in the following branches:

- weapons for armed military and police units, as well as for hunting and sporting purposes,
- components, parts and assemblies for the aircraft and motor industries,
- and special tooling for machine production.

Ceska zbrojovka a.s., Uhersky Brod is a long-standing manufacturer of small firearms. Originally the company was oriented on production of small military arms, however over the time the production has been extended for products for civilian use both for sports and hunt shooting.

Ceska zbrojovka is constantly increasing the volume of production and it is expanding its assortment of small arms, regarding the types and modofications. The significant feature of firearms is their high quality, long-lasting reliability and accuracy. These attributes bring the permanent interest in purchase and use of these products. Currently Ceska zbrojovka represents one of the biggest world manufacturers of small firearms, which is supported by the fact that the company is selling its products into about 100 countries around the world.

The excellent qualities of products from Ceska zbrojovka has created, during the existence of the company, the image of high quality on domestic, as well as on world market. Therefore the company considers as its duty to ensure the best parameters of its products in the future.

Ceska zbrojovka invests each year considerable financial volumes on purchase of top-class technologies, especially in the field of computer numerically controlled machining centers and computing techniques so as to improve the qualities and properties. Thanks to the CAD designing of products, the company is able to react promptly on market demand by the development of new products with perfect properties. Therefore the company launches new products each year.

Ceska zbrojovka is the major owner of daughter companies:

- CZ Export Praha, s.r.o. (100% share),
- CZ-USA Kansas (100% share),
- Union CS, spol. s r.o. (80% share).

The company has concluded the economic year 2006 with the loss of 152 476 thousand CZK. This result is caused by the high impairment of assets’ value in amount of 150 484 thousand CZK. As a positive element of management is the reach of positive value of cash flow and another important indicator of financial stability of company is the fact that Ceska zbrojovka has conducted business without overdue payables during the whole year 2006.

We can indicate the year 2006 as a successful from the point of view of sales reached. The business plan has been fulfilled on 99.8 per cent. The supplies of firearms for Police of the Czech Republic, which were terminated in 2005, were substituted by new contracts. The targeted marketing activity of all senior managers on consigned territories has also had the influence on such result. The success of rifle squad seems to be contributing to increased demand for firearms among sportsmen, as well as in army units.

Investments into participation on international fairs in USA, Germany and regional fairs help to perceive the label CZ, strenghten its value and image in sector of small firearms in the long run.

The business plan of the company for next few years is following:

- To increase the utilization of assets and to reach the profitability in the segment of firearms production.
- To improve the labour organization and to simplify the processes.
- To increase the labour productivity in all sectors.
- Further product quality improvement.
- To support the development of production for automotive industry.
- Higher emphasis of marketing and promotional activities, innovations and development of new products.

Ceska zbrojovka is planning to invest particularly into development of firearms manufacturing, automotive production, protection and improvement of life environment and improvement of labour safety.

The year 2006 has been particularly the year of stabilization. Ceska zbrojovka has made necessary steps for return to balanced economic management and has intention to continue in this trend.

6.3 Application of dicrete-time trinomial model for valuation of company's equity as a real option

In the following sections will be decribed the estimation of input data for calculation of present value of free cash flow, debt, cost of capital and subsequently is described algorithm of calculation of option value, which expresses the value of equity of examined company.

6.3.1 Estimation of free cash flow

The cash flow can characterized as a flow of financial resources (their inflow or outflow) of the company in certain period in connection with its economic activity.

The point of departure is earnings before interest and taxes, so called EBIT. First of all the development of EBIT in the past five years has been examined. For the analysis of past evolution, the trend analysis has been used to determine the tendency of future development. The auxiliary calculations are stated in the table 6.1:

Table 6.1: *Trend line*

Year	***t***	y_t	ty_t	t^2	**est** T_t
2001	1	4 420	4 420	1	65 482
2002	2	145 126	290 252	4	54 383
2003	3	40 880	122 640	9	43 284
2004	4	30 727	122 908	16	32 185
2005	5	-26 444	-132 220	25	21 086
2006	6	31 700	190 200	36	9 987
Average	3,5	37 734,8	99 700	15,2	37 735
Sum	21	226 409	598 200	91	226 409

Using the parameters from the table, the parameters *a* and *b* has been calculated:

$$b = \frac{\sum ty_t - \bar{t}\sum y_t}{\sum t^2 - n\bar{t}^2} = \frac{598\,200 - 3{,}5 \cdot 226\,409}{91 - 6 \cdot 3{,}5^2} = -11\,099\,,$$

$a = \bar{y} - b\bar{t} = 37\,734{,}8 - (-11\,099) \cdot 3{,}5 = 76\,581{,}1$.

The equation of estimated trend line has therefore form of

$est\ T_t = a + bt = 76\,581{,}1 - 11\,099t$.

The calculation of trend line for known years has been made and results are stated in the table above. Now the prediction (extrapolation) of expected high of EBIT for years 2007, 2008, 2009, 2010 and 2010 (to these years correspond values of time variable *t* = 7, *t* = 8, *t* = 9, *t* = 10 and *t* = 11) has to be made.

Into equation of trend line have been supplied these values of *t* and results obtained are following:

Table 6.2: *Prediction of EBIT for following 5 years*

Year	**2007**	**2008**	**2009**	**2010**	**2011**
est T_t	-1 111	-12 210	-23 309	-34 408	-45 507

Subsequently, the analysis of dependence between adjacent members of one time series has to be made. The degree of closeness of this dependence is the autocorrelation coefficient of first order r_1. The auxiliary calculations are stated in the table below:

Table 6.3: *Auxiliary calculations for calculation of autocorrelation coefficient (in millions CZK)*

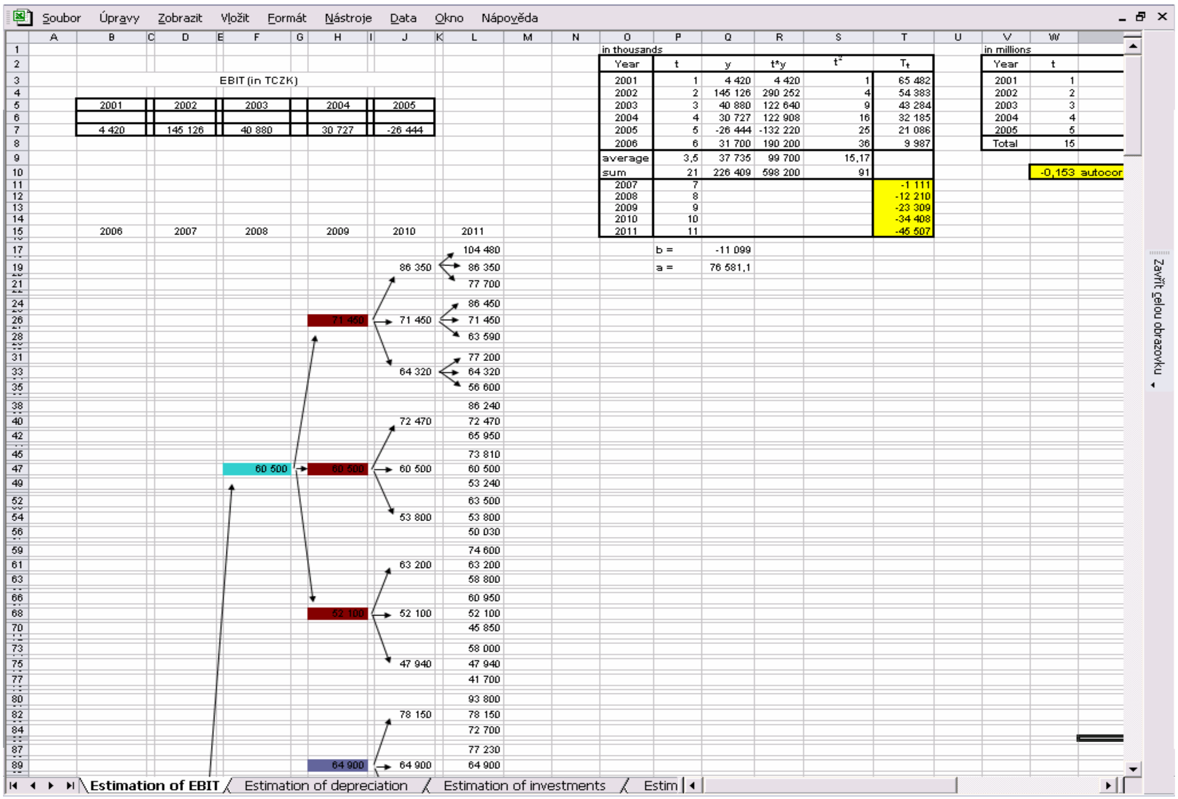
Year	***t***	y_t	y_{t+1}	y_t^2	y_{t+1}^2	$y_t y_{t+1}$
2001	1	4,4	145,1	19,4	21 054	638,4
2002	2	145,1	40,9	21 054	1 672,8	5 934,6
2003	3	40,9	30,7	1 672,8	942,5	1 255,6
2004	4	30,7	-26,4	942,5	697	-810,5
2005	5	-26,4	31,7	697	1 004,9	-836,9
Total	15	194,7	222	24 385,6	25 371,2	6 181,3

When we fill the equation of r_1 with the calculations from the table 6.3, we obtain

$$r_1 = \frac{(n-1)\sum_{t=1}^{n-1} y_t y_{t+1} - \sum_{t=1}^{n-1} y_t \sum_{t=1}^{n-1} y_{t+1}}{\sqrt{\left[(n-1)\sum_{t=1}^{n-1} y_t^2 - \left(\sum_{t=1}^{n-1} y_t\right)^2\right]\left[(n-1)\sum_{t=1}^{n-1} y_{t+1}^2 - \left(\sum_{t=1}^{n-1} y_{t+1}\right)^2\right]}} =$$

$$= \frac{5 \cdot 6181{,}3 - 194{,}7 \cdot 222}{\sqrt{(5 \cdot 24385{,}6 - 194{,}7^2)(5 \cdot 25371{,}2 - 222^2)}} = -0{,}153$$

The autocorrelation coefficient has been calculated as -0,153 which mean that the dependence of single members of the time series is very low and we cannot rely on the trend line. Therefore the prediction of its future development has been made on the basis of expert estimation and on the basis of budget and business plan for future five years, which was obtained from the company's management. On the picture 6.1 is shown the calculation of EBIT in Excel sheet:



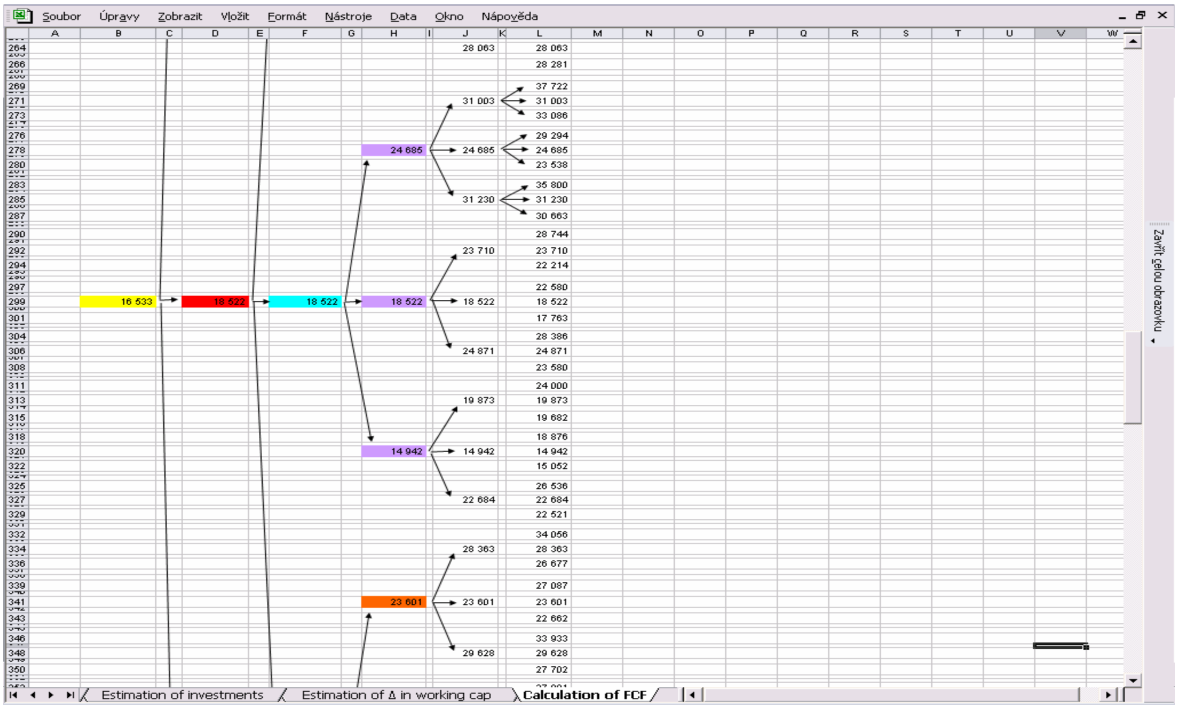
Picture 6.1 *Estimation of trinomial tree for EBIT in Excel sheet*

The trend analysis has been applicated also on the prediction of future development of the depreciation. The procedure of calculation is equal to the calculation for EBIT stated above. Thus we will not state the whole calculation here in this section, as the procedure is part of calculations made in Excel sheet on enclosed CD.

As the autocorrelation coefficient for depreciation has been computed as -0,191, which is again very low and we will not rely on the trend line. The prediction was made on the basis of expert estimation as for EBIT.

Estimation of future investments and change in net working capital has been made on the basis of expert estimation and business plan for future five years. Use of trend analysis or any other method would be unreasonable.

By applying these values into equation (3.7) the estimation of FCFF is made for individual nodes of the trinomial tree, where from each node of the tree can arise three possibilities – increase, decrease or constant development of value in the following period. The tax rate is assumed to be constant at 24 per cent. See calculations in the following picture 6.2:



Picture 6.2 *Estimation of trinomial tree for FCFF in Excel sheet*

6.3.2 Estimation of the weighted average cost of capital (WACC)

Weighted average cost of capital is defined as a weighted average of cost of equity and cost of debt.

WACC = cost of equity (equity / (debt + equity)) + cost of debt (debt / (debt + equity))

Cost of equity is calculated according to CAPM model as

$$R_A = R_F + (E(R_M) - R_F) \cdot \beta_A ,$$

For the calculation was used R_F at rate 3.6 per cent, risk premium at rate 5 per cent and β_A is calculated according to equation (3.9) as 1.036. The cost of equity computed is 8.78 per cent.

Cost of debt has been calculated as a weighted average of various interest rate of various debts of the company. The cost of debt has been determined as 3.04 per cent, which is the real interest rate.

The weighted average cost of capital has been calculated as 6,913 per cent for initial year 2006 and subsequently has been carried out the estimation of this parameter till 2011. All calculations are made in the Excel sheet on the enclosed CD.

On the following picture 6.3 is shown the calculation of WACC in Excel sheet:

in thousands:		
Industry beta (levered)	1,02	
Industry tax rate	0,3147	
Industry D/E	0,42	
Leverage factor	*1,287826*	
Industry unlevered beta	0,7920325	
Unlevered	0,7920325	
CZ Tax rate	24%	
CZUB Debt/Equity	0,4060284	
Leverage factor	*1,3085816*	
Beta levered	**1,0364391**	
Risk free rate	3,60%	rate of return of 15-year Treasury Bonds
Market risk premium	5,00%	judgemental - rate of return of Czech stock market
Calcuated DF (post tax)=cost of equity	8,78%	
Cost of debt (interest rate)	3,04%	
Debt FV	229	only debts bearing interest
Equity FV	564	price per share x number of shares issued (820 x 687 494)
D+E	793	
D/E at FV	41%	
WACC	**6,9133%**	

Picture 6.3 *Calculation of WACC in Excel sheet*

6.3.3 Determination of present value of FCF

It is assumed that the company will exist infinitely long time and therefore the present value of FCFF is determined as a present value of infinite sequence of FCFF discounted at a weighted average cost of capital:

$$PV\ of\ FCFF = \frac{FCFF_t \cdot (1+\pi)^n}{WACC}, \qquad (6.1)$$

where π is the inflation rate. The FCFF is augmented for the inflation rate each year. All calculations are made in the Excel sheet on the enclosed CD.

6.3.4 Estimation of the debt

The initial value of the trinomial tree has been determined as the sum of all loans (which are bearing interest) which are stated on the side of liabilities in the year 2006. The value of the initial node is 229 200 thousand CZK. The estimation of the future development has been made on the basis of expert estimation and business plan provided by the company management for next five years.

6.3.5 Calculation of the intrinsic value of call option

On the basis of values calculated according to the procedures described in previous chapters, the intrinsic value of the call option has been calculated for individual nodes of trinomial tree according to the relation:

$$Intrinsic\ value\ of\ \ call\ option = \max(S - K, 0), \tag{6.2}$$

where S is the present value of the underlying asset (in this case it is the present value of FCFF) and K is the strike price of the option, which in practise means the value of the debt. All calculations are made in the Excel sheet on the enclosed CD.

6.3.6 Calculation of the value of equity as an American call option

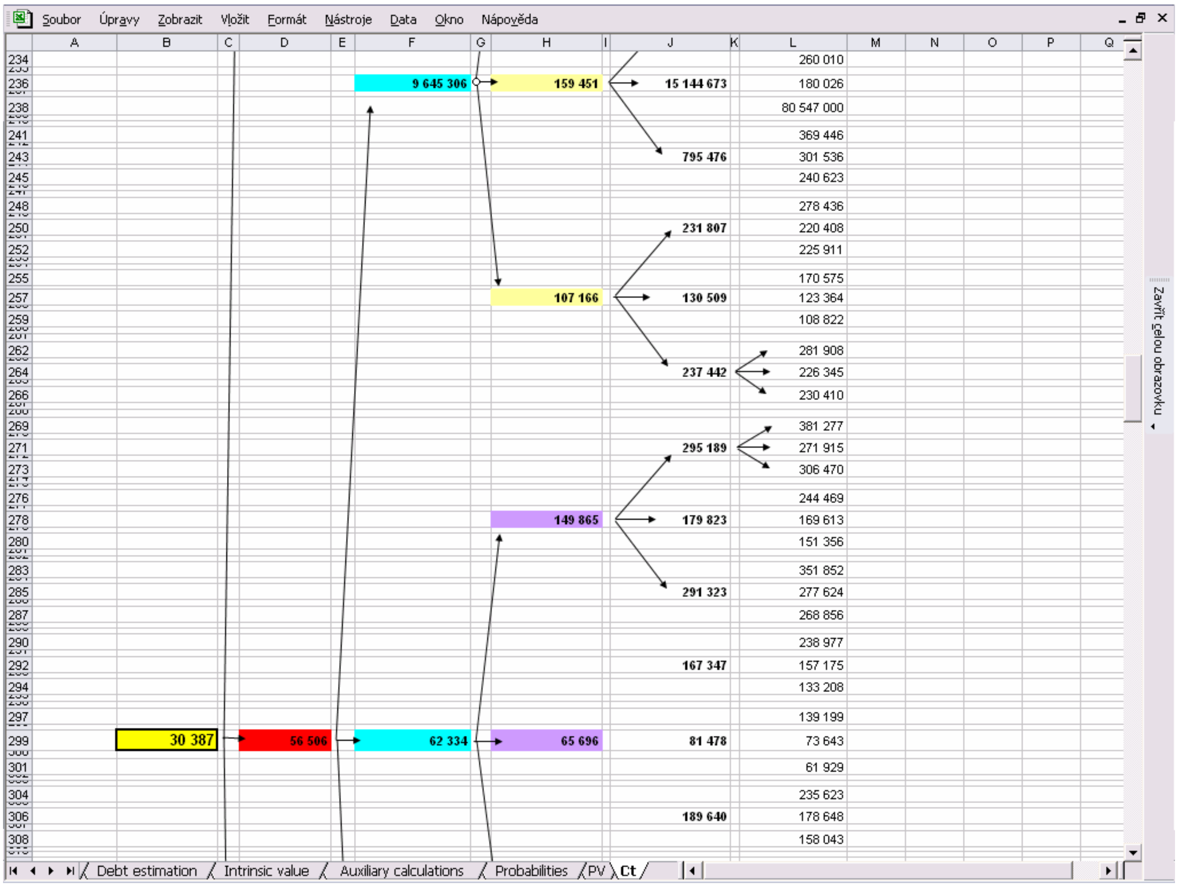
Calculation of the value of the equity as an American call option is determined for individual nodes of trinomial tree as follows:

$$C_t = \max(PV[E(C_{t+1})]; Instrinsic\ value\ of\ \ call\ option), \tag{6.3}$$

where it is proceeded from the end of the trinomial tree. The expression $PV[E(C_{t+1})]$ represents the present value of expected value of the option in subsequent period and is determined as:

$$PV[E(C_{t+1})] = (1+r)^{-1} \cdot [C_{t+1}^{u} \cdot p_i + C_{t+1}^{d} \cdot q_i + (1 - p_i - q_i) \cdot C_{t+1}^{m}]. \tag{6.4}$$

All calculations are made in the Excel sheet on the enclosed CD. In the following picture 6.4 are shown the final calculations and the value of equity as the initial node of trinomial tree:



Picture 6.4 *Value of the equity as the initial node of the trinomial tree in Excel sheet*

7 Discussion and Conclusion

The aim of this thesis was the application of option pricing methodology when valuating certain company from the armament industry.

The chapters 3, 4 and 5 are considered as the literary search. In the third chapter, there were described more common valuation methods, such as discounted free cash flow valuation as a traditional valuation method, relative valuation and valuation of company based on EVA.

Subsequent two chapters (fourth and fifth) were dedicated to the detailed explanation of financial and real options and to the description of basic option pricing methods, such as binomial and trinomial method and Black-Scholes method. This chapter gave an overview of analytical, as well as discret-time valuation models and there were also mentioned basic assumptions for application of these models, their advantages or disadvantages for their practical use.

The sixth chapter has been dedicated to the practical application of the trinomial method for valuation of the selected company operating in the armament industry. According to the algorithms stated in the previous chapters the value of the call option has been calculated, and thus we have obtained the value of the company's equity.

The selected company has been valued on such basis, that the equity has been considered to be an American call option. It means that we consider the possibility that company's owners can decide for its liquidation at any time, and especially when the company's debt is higher than its equity. Unlike the European call option, which can be exercised only at a certain date. Therefore has been chosed the American call option as it enables the flexibility.

In the calculation we have assumed that the WACC will remain inchanged during the whole considered period. The estimation of all inputs for the calculation has been made on assumption that the political and economical situation will remain quite stable. However we have to consider also the possibility of rapid and significant change in the whole industry. The armament industry is quite unstable and it is dependent on many external factors, especially on the political situation in the whole world. The possibility of conflict between United States and Iran is relatively high at the present. Such a conflict would significantly increase the sales of armament companies, including our valued company, which would significantly change the values coming from trinomial tree. The situation in African and South American countries, which are very significant customers of armament companies, is unstable as well. In these countries the threat of embargos is very high. In such situation we should have considered in the calculation with much higher volatility of inputs, which would surely change the value of our company.

On the basis of calculation mentioned in previous chapters and of assumption that the external industry enviroment remains quite stable, the value of equity has been calculated in amount of 30 387 thousand CZK. Subsequently we have to add to this value also the value of debt, which is 229 200 thousand CZK and we thus we obtain the value of the firm in the amount of 259 587 thousand CZK.

For the valuation of the company can be used many others and more traditional methods. Firms usually use for their valuation these traditional methods for their simplicity. However in these methods, such as discount free cash flow valuation, the flexibility of decision making is not distilled. The option pricing method functions on the basis of expectations on capital market and it is able to react in a flexible way on the changing expectations. This may be considered as an advantage of this method.

Nevertheless, the trinomial method has also few disadvantages. These disadvantages are the main reasons why trinomial method is not used so often in real world valuation. The option pricing method is suitable for valuation of companies quoted at the well working stock exchange. The problem arises when the company is not quoted at the stock exchange of the stock market does not function properly. In the case the underlying asset, thus the firm, is not publicly traded, the inputs for the calculation and variance in their value cannot be extracted from financial markets and have to be estimated. Therefore the calculations have much more estimation error than in traditional valuation methods. For such estimation, experts have to consider also factors which were mentioned above – threat of world wars, national wars and conflicts, embargos and many others, which in the case of well functioning stock markets this problem is eliminated. Therefore the estimation of volatility is very demanding and difficult. In addition the process of calculation is very time demanding which is incovenient for practical use.

Although the option pricing method is very complex, it is very suitable for valuation of companies quoted at the well functioning stock exchanges. At its application it is necessary to be aware of the increased demandingness and of the higher probability of estimation error.

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Attachment 1

On the attached CD are posted calculations used in valuation of the company. In the file called 1st Step – Estimation of the FCFF are posted calculations as described in the chapter 6.3.1. In the file 2nd Step – Calculation of value of equity are posted calculations as described in the chapters 6.3.2 – 6.3.6.